# Exploring heterogeneities in health outcomes and space: Integrating quantile and geographically weighted regression

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Tse-Chuan Yang, Ph.D The Social Science Research Institute The Pennsylvania State University 803 Oswald Tower, University Park, PA 16802, USA.

Stephen A. Matthews, Ph.D Department of Sociology The Pennsylvania State University 601 Oswald Tower, University Park, PA 16802, USA

Vivian Yi-Ju Chen, Ph.D Department of Statistics Tamkang University 151 Ying-Chuan Rd, Tamsui, Taipei, Taiwan 251

# Abstract

The past decade has experienced growth in the investigation of heterogeneous associations with independent variables across the distribution of either the dependent variable or across geographic space. The former is implemented using quantile regression (QR), whereas the latter has been popularized by geographically weighted regression (GWR). Demographers have lagged other fields in adopting either of these methods. In this paper, we combine QR and GWR to create an innovative approach to simultaneously explore the heterogeneity embedded in both variables and space – an approach we name geographically weighted quantile regression (GWQR). The goal of this study is to introduce GWQR to demographers. We illustrate GWQR in two studies, the first using ecologic data (US county-level mortality) and the second using individual-level data (obesity in Philadelphia). Significant heterogeneities across space and the distributions of (a) mortality and (b) body mass index suggest that heterogeneity commonly exists and should be considered in model specification.

# Introduction

Most population studies seek to identify the associations between a dependent variable and a set of independent variables. From a statistical perspective, these associations should be regarded as random variables. The typical multivariate regression techniques, however, depict these associations using measures of central tendency (i.e., mean, median and mode) of the distribution of a dependent variable (Hao and Naiman 2007). Focusing on the mean value is arguably the most common approach to summarizing how a dependent variable responds to change in an independent variable (Kleinbaum et al. 2007). While the mean regression is able to provide a parsimonious picture of these associations of interest, in applications, its underlying assumptions, i.e., homogeneity and independence, are often violated, especially with social science and geospatial data (Cressie 1991; Hao and Naiman 2007). These analytical issues have been largely ignored in empirical studies and we suggest that the way to move forward is to explore two sources of heterogeneity in data – the statistical heterogeneity of a dependent variable and the spatial heterogeneity among observed units.

To better understand the concept of heterogeneity, it is imperative to first introduce its opposite, homogeneity. In statistics, homogeneity refers to the level of similarity in distributions and the definition of similarity ranges from a single attribute (e.g., variance or mean) to total sameness. For example, in ordinary least squares (OLS) regression, the residuals are assumed to have a homogeneous variance, which indicates the variance does not vary across subsamples (Sokal and Rohlf 1981). Extending homogeneity to a spatial context, the relationships of interest that do not vary with the spatial dimension can be described as homogeneous. For simplicity and convenience, both statistical and spatial homogeneity are widely assumed in most conventional

analytic techniques and heterogeneity per se is ignored or regarded as an unwelcome complication (Kleinbaum et al. 2007; Pickett and Cadenasso 1995).

Heterogeneity can be defined as the lack of homogeneity. In other words, this is when the relationships between the mean of a dependent variable and a certain independent variable cannot be generalized to other locations along the distribution, especially when the data are skewed or distributed in the tails (Hao and Naiman 2007). The development of quantile regression (QR) has provided a method that examines how predictors are associated with a dependent variable (Koenker 2005; Koenker and Bassett 1978). More and more empirical studies are using QR to provide a nuanced insight into heterogeneity (Abrevaya and Dahl 2008; Austin et al. 2005; Han, Powell, and Pugach 2011). QR is preferable to traditional analytic approaches (e.g., OLS) due to its flexibility in dealing with non-normally distributed errors, robustness against outliers, and ability to detect heterogeneity (Koenker 2005). As of yet, few demographers have taken advantage of QR to tackle heterogeneity. For a recent exception please see Yang et al. (Forthcoming).

In the past decade, the resurgence of spatial demography can be attributed to the rapid growth in geospatial data and new spatial analytical methods (Anselin 1988b; Voss 2007). However, spatial econometrics models, most commonly used spatial analytic methods, assume that the associations between the independent and dependent variables are homogeneous across space; i.e., do not vary by location (Fotheringham 1997). Interest in the homogeneity assumption stimulated the development of geographically weighted regression (GWR), an exploratory method that investigates spatial heterogeneity, allows researchers to better understand the underlying spatial process between independent and dependent variables, and can be used to help refine model specification (Brunsdon, Fotheringham, and Charlton 1998b; Fischer et al. 2010; Foody 2003; Fotheringham, Brunsdon, and Charlton 2002; Wheeler 2007). To date, relatively few studies have employed GWR to address spatial heterogeneity in demography and health research (Chen et al. 2010; Partridge and Rickman 2007; Shoff, Yang, and Matthews Forthcoming; Yang et al. 2009).

In contrast to other disciplines such as forestry, geography, economics, and environmental science, demography has lagged in exploring the heterogeneity embedded in both variables (via QR) and/or space (via GWR). We argue that it is time for demographers to not only catch up with the trend that embraces statistical heterogeneity, but also in the examples we develop, consider spatial heterogeneity in demographic research.

This paper is organized as follows. First, we elaborate on the limitations of conventional regression approaches, model improvements offered by both QR and GWR, and discuss in more detail statistical and spatial heterogeneity. We then introduce a newly developed spatial analysis tool, geographically weighted quantile regression (GWQR). We provide two empirical examples to demonstrate the potential utility of GWQR. In the first example, we draw on an ecological study of county-level mortality across the US, and in the second example, we use an individual level study of obesity within a small area within Philadelphia. Next, the preliminary findings and lessons learned from the two empirical examples will be summarized. Finally, the paper will conclude with a discussion of future directions in methods to explore in demographic research.

#### **Literature Review**

#### Limitations of traditional regression approach

Linear regression is arguably the most popular analytic tool used to answer the question of how a dependent variable is associated with a set of predictors in health and population studies (Kleinbaum et al. 2007). There are, however, several well-know disadvantages of this analytic approach. For example, the independence assumption (i.e., data are randomly distributed) is often violated in ecologic studies (Cressie 1991), and the normality assumption (i.e., data follow the Gaussian distribution) also may not hold especially when a dependent variable has a heavytailed distribution (Hao and Naiman 2007). Failing to meet these modeling assumptions may result in biased estimates and misleading conclusions. To address these issues, new statistical methods have been developed (see below).

Ecological social and health data have been found to be spatially dependent and the independence assumption has been challenged by Tobler's first law of geography (Tobler 1970). Spatial dependence can be found in both dependent and independent variables as well as in model residuals. Spatial econometrics approach has been developed to handle these issues and has been widely used. In general, this approach takes spatial dependence into account by including a spatial lag effect of the dependent variable and/or a spatial error term in the residuals (LeSage and Pace 2009). However, spatial econometric methods focused on spatial dependence and most other spatial models are designated to estimate a single, or *global*, regression equation based on spatial data where an underlying assumption is that the relationships between the independent variable are homogeneous (e.g., stationary) over space (Fotheringham 1997).

There are good reasons to question the homogeneity assumption in conventional spatial analysis (Brunsdon, Fotheringham, and Charlton 1998a; Fotheringham 1997). First, random sampling variations will inevitably contribute to the observed spatial associations. While this source of spatial heterogeneity is not of interest for most researchers, it may complicate significance testing. Second, some relationships between the dependent and independent variables intrinsically vary across space. For instance, the spatial variations in individuals' attitude toward the health care system (i.e. levels of distrust) may produce different responses to the health outcomes over space (Yang and Matthews 2012). The idea that human behaviors differ by places echoes the recent work on the impacts of locality and residential neighborhood environments on health (Diez Roux and Mair ; Macintyre, Ellaway, and Cummins 2002). Third, from a modeling perspective, *global* modeling may be a misspecification of reality and the variables included in the models may not be represented by the correct function form (Fotheringham 1997). A global model is useful, but the model specification is not sufficient to detect evidence of spatial heterogeneity (Fotheringham, Brunsdon, and Charlton 1997, 2002).

The traditional regression approaches concentrate on the measures of central tendency of the distribution of a dependent variable. This focus on central locations means that, at best, the results only can be presumed to be homogeneous across the dependent variable's distribution. This feature may have inadvertently steered demographers, social, and health scientists away from questions relevant to non-central locations (Hao and Naiman 2007). For example, understanding the relationship between wage inequality and educational attainment in the tails of the wage distribution may be more helpful than models estimated for the central locations (Machado and Mata 2005). Similarly, the associations of maternal characteristics with birth weights should be more important in the low end (among low birth weight infants) than any other locations of the birth weight distribution (Abrevaya and Dahl 2008). The traditional approach inherently fails to characterize the relationship between a dependent variable's *distribution* and predictors, and the question of how changes in predictors affect the shape of a dependent variable's distribution remains unanswered (Koenker 2005).

While alternative approaches have been developed to overcome some methodological concerns (e.g., median regression to minimize the influence of heavy-tailed distribution), the QR

approach, which focuses on the whole distribution of a dependent, has been popular in economics (Koenker 2000), but has only recently emerged in health and demographic research (Austin et al. 2005; Yang et al. Forthcoming). QR divides the whole distribution into quantiles and estimates the conditional quantiles as functions of explanatory variables. Quantile is a term that generalizes specific locations, such as quartile, quintile, decile, and percentile, and it can represent any predetermined locations of a distribution (Koenker and Bassett 1978). Since any quantiles can be modeled, researchers can choose any positions in a distribution to tailor specific research questions; thus, it becomes possible to generate a graphical profile of how a dependent variable's distribution is affected by predictors.

While GWR and QR, respectively, address some of the limitations of the traditional regression approaches, and important shared featured of these two methods is the focus on the homogeneous assumption. GWR dissects the global spatial process into multiple local processes that are allowed to be heterogeneous across the research area and QR enables the examination of whether the associations between a dependent variable and predictors vary across a dependent variable's distribution. Both spatial and statistical heterogeneities have been shown to generate new knowledge and inform place- and population-specific policies (Fotheringham, Brunsdon, and Charlton 2002; Hao and Naiman 2007).

#### Heterogeneity

#### Statistical heterogeneity, heteroscedasticity, and non-stationarity

From a statistical viewpoint, Dutilleul and Legendre (1993) suggested that "there are as many definitions of heterogeneity as there are parameters for a statistical distribution, each of these definitions related to the object to which heterogeneity applies (p.155)." At its most simple, heterogeneity is the opposite of homogeneity. However, to better understand what statistical heterogeneity means, several terminologies derived from the concept of heterogeneity need to be clarified.

In the traditional regression modeling, heteroscedasticity refers to the equality of variances (Dutilleul and Legendre 1993; Glaser 1983; Sokal and Rohlf 1981), and has been found to be one consequence of heterogeneity. As heterogeneity may refer to the equality of more than one distributional feature, statisticians have expanded the concept of heteroscedasticity by taking the mean value of the variable of interest into account and proposed the idea of non-stationarity (Dutilleul and Legendre 1993). For a given statistical characteristic, the level of dissimilarity (similarity) between two samples may be used to define the level of heterogeneity (homogeneity).

By extension, statistical stationarity can be classified as strict, weak, and intrinsic stationarity (Banerjee, Carlin, and Gelfand 2004). Strict stationarity assumes that any changes in the observations would not affect the variance or mean values. Weak stationarity only requires that both the mean and variance remain constants regardless of the changes in data. Intrinsic stationarity stands as long as the changes in data follows a distribution with a zero mean and a finite variance (Banerjee, Carlin, and Gelfand 2004; Dutilleul and Legendre 1993; Lloyd 2011). The discussions above indicated that heterogeneity is the largest construct that covers both heteoscedasticity and non-stationarity. These two concepts are specific to certain distributional features and should be used carefully. While these definitions related to heterogeneity emerge from a statistical perspective, they have been used to define spatial stationairty in spatial data analysis (see next section).

Spatial heterogeneity and its significance

When applying heterogeneity to a spatial context, it is crucial to distinguish the geostatistical pattern from a spatially continuous process (Dutilleul and Legendre 1993). The former concerns the densities of points (observations) across space, whereas the latter represents the variability across sub-areas with respect to the relationships among variables. The geostatistical pattern process is interested in merely the locations of observations and defines spatial heterogeneity as the significant difference in density variations among sub-regions within an area. For example, whether the patients with a certain type of disease are observed randomly across space would be the major concern from the geostatistical aspect. Once the densities of patients (after taking population at risk into account) are found to be unevenly distributed in a given area, one could conclude that spatial heterogeneity is present (Dutilleul and Legendre 1993). The Poisson process has been widely used in this paradigm to examine whether the spatial distributions of points are random and both spatial over- and under-dispersion could contribute to spatial heterogeneity (Lloyd 2011).

Spatial continuous surface process focuses on the relationships among the features that are attached to observations. Spatial heterogeneity refers to the situation that the observed values either for a given variable or among variables changes spatially, with some areas having greater (or smaller) values than others. A recent study, for example, demonstrated that the relationship between health care system distrust and self-rated health among the elderly varied significantly within the Philadelphia metropolitan area (Yang and Matthews 2012). Unlike the geostatistical process, the surface process has focused on whether there is a spatial pattern regarding the observed values of interest and what the pattern looks like (McIntosh 1991). More importantly, this type of spatial heterogeneity has been found to be subject to scale (i.e., measurement units), where a decrease (increase) in the geographical size of measurement unit may convert heterogeneity into homogeneity (homogeneity into heterogeneity). This issue calls for careful choices of analytic units, thoughtful interpretation of spatial heterogeneity, and a clear understanding of local spatial processes (Dutilleul and Legendre 1993).

The concept of statistical stationarity has been loosely applied to spatial analysis domain and should be further clarified. According to the earlier discussion, stationarity refers to the situation that any changes in the data would not change certain distributional features of interest. Cressie (1991) define "changes in the data" as the change in the locations of observations to explore spatial stationarity in spatial point modeling. However, by contrast, Brunsdon and colleagues (Brunsdon, Fotheringham, and Charlton 1998a; Brunsdon, Fotheringham, and Charlton 1998b) described spatial stationarity as a phenomenon where the associations between dependent and independent variables do not change by geographical locations. There is no agreement on which definition is correct but it seems that the former closely mirrors the concept of statistical stationarity and the latter better fits the concept of heterogeneity in spatial continuous process.

How does spatial heterogeneity inform demographic and health research? At its most basic, we are challenging the conventional assumption that the same stimulus promotes the same response. We believe this to be so for several reasons. First, heterogeneity should be regarded as an inherent feature of a society (Dutilleul and Legendre 1993). Many social characteristics are unevenly distributed and people are engaged in different geographic scales that affect spatial heterogeneity (Matthews 2011). Indeed, it is naïve to assume that the relationships of interest are constant everywhere (Lloyd 2011). Second, spatial heterogeneity may reflect the population dynamics with the natural and/or social environment that have not been considered by researchers. This unobserved association may result in heterogeneous relationships across regions and encourage scholars to think with a local perspective and locate areas that need special attention (Anselin 1988a; Dutilleul and Legendre 1993). Finally, the spatial pattern generated by heterogeneity provides valuable information to those researchers attempting to identify factors that are predictive of the spatial pattern (Fotheringham, Brunsdon, and Charlton 2002; Lloyd 2011). This focus on spatial heterogeneity echoes a re-emergent interest in spatial inequality (Lobao, Hooks, and Tickamyer 2007).

## Putting heterogeneities together

The discussion above suggests that to a certain degree heterogeneity is a product of the perspective and analytical decisions of the researcher. In this paper, we are focused on the associations between a dependent variable across the statistical distribution and across geographic space. Currently, QR and GWR are two methods that are based on the non-parametric estimation approach and designated to separately explore statistical and spatial heterogeneities (Fotheringham, Brunsdon, and Charlton 2002; Koenker 2005). Only recently have researchers considered statistical and spatial heterogeneity within the same methodological framework (Chen et al. 2012; Reich, Fuentes, and Dunson 2011).

GWQR has been developed by integrating QR with GWR, creating a synergy that makes it possible to simultaneously account for the heterogeneities across space and the distribution of a dependent variable (Chen et al. 2012). The novel analytic approach allows researchers to dissect the global processes into local processes and investigate whether and how the relationships of predictors with the dependent variable differ across the distribution of the dependent variable. The original paper (Chen et al. 2012) covers the technical explanations in more detail than space permits here.

#### Methodology – Non-parametric GWQR Framework

The GWQR framework is built upon the GWR approach and we are thus focused on GWR and refers readers elsewhere for the details of QR (Hao and Naiman 2007). Following Fotheringham and colleagues (2002), a Gaussian GWR model can be expressed as:

$$Y_i = X_i^{t} \beta(u_i, v_i) + \varepsilon_i = \beta_0(u_i, v_i) + \sum_{k=1}^{p} X_{ik} \beta_k(u_i, v_i) + \varepsilon_i$$
(1)

where  $Y_{i}$ , i = 1, ..., n, represents the response observations collected from location i in space and the corresponding geospatial covariate vector can be written as  $X_i = (1, X_{i1}, X_{i2}, ..., X_{ip})^t$  of dimension (p+1) including the constant 1 for intercept.  $\beta(u_i, v_i) = \{\beta_0(u_i, v_i), \beta_1(u_i, v_i), ..., \beta_p(u_i, v_i)\}$ indicates the local coefficient estimates,  $(u_i, v_i)$  represents the coordinates of location i in space, and  $\varepsilon_i$  is the error term with a mean of zero and common variance  $\sigma^2$ . The coefficient estimations,  $\beta_k$ , are specific to each location i, which yields mappable local statistics.

One feature of GWR is the ability to use a kernel-based density function centered on each observation to estimate the local parameters. That is, for a given location  $(u_0, v_0)$ , the  $\beta$ 's are locally computed by minimizing:

$$\sum_{i=1}^{n} \left[ Y_i - \beta_0(u_i, v_i) - \sum_{k=1}^{p} X_{ik} \beta_k(u_i, v_i) \right]^2 K(\frac{d_{i0}}{h})$$
(2)

where *K* is a kernel function and *h* is the bandwidth, which controls the smoothness of the estimates.  $K(\frac{d_{i0}}{h})$  indicates the geographical weight assigned to observation  $(X_i, Y_i)$  and depends on the distance  $d_{i0}$  between the given location  $(u_0, v_0)$  and the *i*th designed location  $(u_i, v_i)$ . Overall, more weight is placed on observations closer to  $(u_0, v_0)$  than those farther away.

The GWQR extends equation (1) by incorporating the features of QR into the response observations  $Y_i$ . Assume that we have a collection of covariate observation vectors  $\{X_i\}_{i=1}^n$  and

the responses  $\{Y_i\}_{i=1}^n$  at different locations  $(u_i, v_i)$  over a study region, the GWQR equation can be expressed as:

$$Y_{i} = X_{i}^{t}\beta^{\tau}(u_{i}, v_{i}) + \varepsilon_{i}^{\tau} = \beta_{0}^{\tau}(u_{i}, v_{i}) + \sum_{k=1}^{p} X_{ik}\beta_{k}^{\tau}(u_{i}, v_{i}) + \varepsilon_{i}^{\tau}$$
(3)

where the vector  $\beta^{\tau}(u_i, v_i) = \left\{ \beta_0^{\tau}(u_i, v_i), \beta_1^{\tau}(u_i, v_i), \dots, \beta_p^{\tau}(u_i, v_i) \right\}^{t}$  are smoothed coefficients at different quantiles (0<  $\tau$  < 1) that must be computed at each location  $(u_i, v_i)$ .  $\varepsilon_i^{\tau}$  is the random error whose  $\tau$  th quantile is conditional on  $X_i$  equals zero. Equation (3) implies that the  $\tau$  th conditional quantile function of  $Y_i$ , given observation vector  $X_i$  for location i with coordinates  $(u_i, v_i), q_{\tau}(X_i, u_i, v_i) \equiv \inf \{y : F(y \mid X_i, u_i, v_i) \ge \tau\}$  should be:

$$q_{\tau}(\mathbf{X}_{i}, u_{i}, v_{i}) = \mathbf{X}_{i}^{t} \beta^{t}(u_{i}, v_{i}) = \beta_{0}^{\tau}(u_{i}, v_{i}) + \sum_{k=1}^{p} X_{ik} \beta_{k}^{\tau}(u_{i}, v_{i})$$
(4)

In contrast to the original GWR equation (1), the parameter estimates of GWQR,  $\beta^{\tau}(u_i, v_i)$ , become the function of location coordinates and are  $\tau$  -dependent. The coefficients,  $\beta_k^{\tau}(u, v)$ , are estimated non-parametrically using kernel smoothing methods. Suppose that each of the coefficients  $\beta_k^{\tau}(u, v)$  has second continuous partial derivatives with respect to the coordinates *u* and *v*, then for  $(u_i, v_i)$  in the neighborhood of a given regression point  $(u_0, v_0)$ , one has, based on Taylor's theorem,

$$\beta_{k}^{\tau}(u_{i}, v_{i}) \approx a_{k}^{\tau} + b_{k}^{\tau}(u_{i} - u_{0}) + c_{k}^{\tau}(v_{i} - v_{0})$$
(5)

where  $a_k^{\tau} = \beta_k^{\tau}(u_0, v_0)$ ,  $b_k^{\tau} = \frac{\partial}{\partial u} \beta_k^{\tau}(u, v) |_{(u,v)=(u_0,v_0)}$ ,  $c_k^{\tau} = \frac{\partial}{\partial v} \beta_k^{\tau}(u, v) |_{(u,v)=(u_0,v_0)}$ . Using local modeling

and with the approximate expression of equation (5), we let

 $\boldsymbol{\theta}^{\tau}(\boldsymbol{u}_{0},\boldsymbol{v}_{0}) = \left[\boldsymbol{a}_{0}^{\tau},\cdots,\boldsymbol{a}_{p}^{\tau},\boldsymbol{b}_{0}^{\tau},\cdots,\boldsymbol{b}_{p}^{\tau},\boldsymbol{c}_{0}^{\tau},\cdots,\boldsymbol{c}_{p}^{\tau}\right] \text{ and choose } \boldsymbol{\theta}^{\tau}(\boldsymbol{u}_{0},\boldsymbol{v}_{0}) \text{ to minimize the following locally weighted function:}$ 

$$\sum_{i=1}^{n} \rho_{\tau} (Y_{i} - \tilde{X}_{i}^{t} \theta^{\tau} (u_{0}, v_{0})) K(\frac{d_{i0}}{h})$$
(6)

where  $\tilde{X}_{i} = [1, X_{i1}, ..., X_{ip}, (u_{i} - u_{0}), X_{i1}(u_{i} - u_{0}), ..., X_{ip1}(u_{i} - u_{0}), (v_{i} - v_{0}), X_{i1}(u_{i} - u_{0}) ..., X_{ip}(v_{i} - v_{0})]^{t}$ . Based on Taylor's theorem, we take the solution of  $a_{k}^{\tau}, k = 0, 1, ..., p$  in  $\theta^{\tau}(u_{0}, v_{0})$  to equation (6), denoted by  $\{\hat{a}_{k}^{\tau}\}$ , as the local linear estimators for  $\{\beta_{k}^{\tau}(u_{0}, v_{0})\}$ . If one chooses to minimize the following loss function

$$\sum_{i=1}^{n} \rho_{\tau} (Y_{i} - a_{0}^{\tau} - \sum_{k=1}^{p} X_{ik} a_{k}^{\tau}) K(\frac{d_{i0}}{h})$$
(7)

then, the resultant minimizer  $\{\hat{a}_{k}^{r}\}\$  serves as the local constant estimator of  $\{\beta_{k}^{r}(u_{0},v_{0})\}\$ . It is known from the theory of QR that the solution of  $\theta^{r}(u_{0},v_{0})$ , which minimizes the loss function (6) has no explicit form. The weighted QR problem of equation (6) can be equivalently reformulated as a problem of optimization in terms of linear programming (Chen and Wei 2005; Koenker 2005). Employing the same approach described here, the estimates  $\hat{\beta}_{k}^{r}(u_{i},v_{i})$  of each QR coefficient  $\beta_{k}^{r}(u_{i},v_{i})$  in equation (3) can be derived by taking  $(u_{0},v_{0}) = (u_{i},v_{i})$ , (i = 1,2,...,n). We have developed a series of macro programs in SAS to implement the GWQR analysis and these programming codes are available upon request.

#### **Examples**

In the remainder of this paper we seek to go beyond the original methodological paper with a more explicit focus on two quite different empirical applications, which vary on sample sizes (from 377 to 3072), for different types of analytical units (ecologic and individual), and for study areas ranging from just a few square miles (inner city Philadelphia) up to the nation (lower 48 states). These empirical demonstrations serve to show the versatility of GWQR and its applicability to demographic and health research. We briefly describe the data and offer basic descriptive statistics below:

*Case 1 – A study of US County-level Mortality:* We used the latest Compressed Mortality Files (CMF) maintained by the National Center for Health Statistics (NCHS) and calculated the 2004-2008 five year average mortality rates as the dependent variable for the contiguous US counties (N=3,072) (NCHS 2011). The average mortality rates were standardized with the 2006 US age-sex population structure. The independent variables include the deprivation index (Townsend, Phillimore, and Beattie 1988), poverty rates, and metropolitan status. The deprivation index was developed for health disparity research and comprised the following four variables: percent of economically active people unemployed (X1), percent of households with more than one person per room (X2), percent of household without vehicles (X3), and percent of housing units that are renter-occupied (X4). To obtain the deprivation index, these variables need to be transformed and standardized as follows (Townsend, Phillimore, and Beattie 1988): let D1=log(X1+1), D2=log(X2+1), D3=X3, and D4=X4. These variables were then standardized into z-scores and in the final step summed into a single deprivation index. Counties with larger values on the deprivation index could be interpreted as more deprived areas. Poverty rates are defined as the proportion of individuals whose income level is below poverty line. Metropolitan counties (coded 1) include those counties that have one or more urbanized areas with more than 50,000 residents and those counties with a total urbanized area of 100,000 population and contiguous counties with strong economic ties. The data for the independent variables were derived from the American Community Survey (ACS) 2005-2009 5-year estimates (US Census Bureau 2010). The distributions of these variables were summarized in Table 1 (Panel A).

[Table 1 here]

#### Case II – A Study of Body Mass Index (BMI) in African American Women in

*Philadelphia:* The Neighborhood Food Environment, Diet, and Health Study is a multi-wave project investigating the role of the built environments on dietary habits. Two neighborhoods, one intervention and one comparison, were matched by baseline retail structure, median income, and racial/ethnic composition. For this empirical example, we focus on the 377 African American women living in the intervention site at baseline. The dependent variable is the women's BMI calculated from self-reported height and weight. Four independent variables are included in the analysis. Self-rated health is a five scale ordinal measure ranging from 1 (excellent) to 5 (poor). Age is treated as a continuous variable. Individual income was classified into 12 ordered groups where 1 indicates an annual income less than \$5,000 and 12 represents an income greater than \$100,000. The fourth and final variable used in this simple example is the distance from a woman's home to the store where she purchased fruit and vegetables. Table 1 (Panel B) provides the descriptive statistics of these variables.

#### **GWQR Results**

Since GWQR is able to estimate the relationships between a dependent variable and predictors at any quantiles of a dependent variable's distribution, the analysis can generate abundant information for each observation at each quantile. Following Fotheringham et al. (2002) for GWR and Koenker (2005) for QR, we reproduce output focusing on the five-number summary statistics to illustrate the variations in local estimates at the following selected quantiles: 5<sup>th</sup>, 25<sup>th</sup>, 50<sup>th</sup>, 75<sup>th</sup>, and 95<sup>th</sup>. Moreover, to better interpret spatial heterogeneity, we visualized spatial patterns with mapping approach developed by Matthew and Yang (2012). Given the space constraint and the goal of our analysis, our discussions are brief and we limit the

mapping to just one key independent variable in each example. The detailed GWQR analytic results are available upon request.

# Case I: Mortality

Table 2 presents the results for the US mortality example. We reported the results at five different quantiles. At each quantile and for each independent variable, the five summary statistics are listed to demonstrate the distribution of the estimated local associations with mortality, which helps us to compare the results across space and quantiles. Several findings were notable. First, the bandwidths utilized in our model tend to be larger toward the upper and lower ends of the county-level mortality distribution, and the bandwidths appear to be relatively symmetrical across percentiles centered on the 50<sup>th</sup> percentile. This bandwidth pattern may reflect the fact that the mortality rates were concentrated around the mean value, as revealed by the low standard deviation (see Table 1, Panel A). In such a case GWQR needs to draw on more observations (i.e., counties) in the kernel density function for reliable estimations at the extremes. Second, the largest bandwidth in Table 2 was 694, which is roughly 23 percent of the total observations (3,072). The low proportion indicates that the spatial processes between mortality and the independent variables, in our example, are fairly local. This implies that the traditional global regression approach is a misspecification. Third, using a conventional method that compares interquartile ranges with global standard errors (Fotheringham, Brunsdon, and Charlton 2002), spatial heterogeneity is identified at each quantile, which may suggest that spatial heterogeneity is not a rare phenomenon, at least, in mortality research. Fourth, comparing the local estimates across quantiles, we also found that the relationships of mortality with the explanatory variables were heterogeneous. For example, the association between the deprivation index and mortality tended to be stronger around the 25<sup>th</sup> and 75<sup>th</sup> percentile than in any other

percentiles, and the relationship of poverty with mortality steadily increases from the low to the high end of the mortality distribution.

# [Table 2 Here]

Figure 1 demonstrates the heterogeneous associations between the deprivation index and mortality across quantiles. The colored areas are the areas where the associations are statistically significant at the p<0.05 level. One consistent pattern across models is that the northern Great Plains (e.g., North and South Dakota, Montana, and Wyoming) are subject to the impact of deprivation. In those counties, high deprivation index is associated with high mortality. The positive association between deprivation and mortality seems to hold along the Mississippi River, especially between the 25<sup>th</sup> and 75<sup>th</sup> percentile. By contrast, an unexpected (negative) relationship between deprivation and mortality was found in the New England area and the US/Mexico border. One plausible explanation for this finding may be related to the Hispanic Paradox, a phenomenon that Hispanic population experience low mortality despite their relatively low socioeconomic status. It should be noted that in this example, we did not include other important variables (e.g., racial composition) in the analysis and thus results could change with the inclusion of new independent variables. As expected, the relationship between deprivation and mortality was heterogeneous and, based on this simple model, deprivation may not be a major determinant of mortality in most of the US.

#### [Figure 1 Here]

#### Case II: BMI in Philadelphia

The second empirical example draws on individual-level data collected in a low-income, predominantly African American neighborhood. GWQR was used to explore heterogeneity in BMI in 377 African American women and the results were presented in Table 3. Before describing the results we again note that the bandwidths utilized in GWQR are larger toward the upper and lower ends of the BMI distribution. We note that the bandwidth at the 50<sup>th</sup> percentile draws on 78 percent of the sample, while at the 75<sup>th</sup> percentile and 95<sup>th</sup> percentiles 95 percent and 84 percent of the sample are used in estimating the weighted local models. These are much larger percentages than in the mortality example, but such high levels are not unusual in GWR applications (Fotheringham, Brunsdon, and Charlton 2002). From Table 1 (Panel B) we know that mean BMI values are high (30, see Table 1) and that the range of reported values is quite wide (we did not remove extreme values as QR and GWQR are robust in handling outliers). On closer examination of BMI the variance-to-mean ratio (1.717) implies that the variable is over-dispersed, possibly due to the presence of clusters of data values.

## [Table 3 Here]

Comparing the interquartile ranges with global standard errors (Table 3) confirms the presence of spatial heterogeneity but the patterning across quantiles varies by predictor. Self-rated health reveals spatial heterogeneity for all but one of the quantiles (the  $25^{th}$  percentile). The GWQR facilitates the comparison of relationships between BMI and the predictors across quantiles; again not surprisingly there is variation and even some switching of sign. For example, focusing on the median values of the distributions of local coefficients, the relationship between BMI and self-rated health is relatively small and negative (median = -0.076) in the lowest quantile (5<sup>th</sup>) but much larger at the 50<sup>th</sup> quantile and above (ranging between 1.517 to -2.485). Distance to the fruit and vegetable store has a positive association at with BMI in the lowest quantiles but the relationship is negative at and above the  $25^{th}$  quantile.

Figure 2 illustrates how the association between self-rated health and BMI varies across quantiles and the study area. From Table 3, we know that the association between BMI and self-

rated health was stable at the 25<sup>th</sup> quantile but otherwise was spatially heterogeneous. The map for the 25 quantile reveals a varying spatial pattern for the coefficient for self-rated health but overall the variation in the relationship at that quantile does not significantly differ from the global modeling and hence homogeneous. The most dominant pattern across the quantile maps is the high positive coefficient in the north-east part of the study area across most quantiles. This persistent pattern could be understood as follows. One is that a large grocery store is just outside the significant area and the respondents in that quadrant are closer than any others in the study area, which contributes to the spatial pattern. The other is that this example only focused on African American but the significant area also has high concentration of Hispanic population. Our GWQR result may imply the community-level influence that is not included in the analysis.

#### [Figure 2 Here]

While this example did not identify spatial heterogeneity to be as serious as in the mortality case, it is none the less an appropriate application of GWQR as knowledge of the lack of spatial heterogeneity is also important. Indeed, from a policy or intervention standpoint knowing what is spatially homogenous may be as important as knowing what is spatially heterogeneous. In the BMI case, the association with self-rated health is spatially heterogeneous, and to a lesser extent so is the association with distance but for age and income the relationships to BMI are relatively stable across the study area. By contrast, the mortality case suggested that heterogeneities deserve further consideration to explain the residential mortality disparity.

# **Discussions and Conclusions**

Statistical heterogeneity has been found to frequently occur when data are not normally distributed and it may provide detailed information relevant to questions across the range of a variable's distribution (e.g., BMI and mortality). In the case of mortality, revealing the

statistically heterogeneous associations can help demographers answer questions regarding the geographical mortality differentials and better focus on the factors that may reduce mortality for specific mortality groups (e.g., the highest quartile). The GWQR analysis offered strong evidence challenging a global model perspective and suggested that the one-model-fits-all approach sacrifices the opportunities to identify spatial heterogeneity and uncover new knowledge. For example, without the GWQR approach, the finding of a negative relationship between the deprivation index and mortality in New England and along the US/Mexico border would not be clearly identified.

While the two empirical examples differed in terms of the sample size, the geographical unit (county/individual), the distances between observations and their elative densities, and the size of the study areas (the contiguous US/20 census tracts), we identified spatial heterogeneity in both studies. Spatial heterogeneity may not be as rare as we might think in demographic and health studies and the homogeneous assumption regarding spatial processes may need to be revisited (Fotheringham, Brunsdon, and Charlton 2002).

One goal of this manuscript is to complement the original GWQR methodology paper and verify the applicability of this new method in different areas of demography. In our first example, we parallel the mortality application of Chen et al. (2012) using updated mortality data and a new set of explanatory variables drawn from the ACS. Both spatial and statistical heterogeneity were found in the new models. This indicates that the spatial heterogeneity found by Chen et al (2012) cannot be attributed to data selection bias. One point we would like to emphasize is that spatial heterogeneity appears to commonly exist in social indicators. In our mortality example, the deprivation index, poverty and metropolitan status all showed heterogeneous relationships with mortality (maps are available upon request). In addition, in our second empirical example, we demonstrated the application of GWQR to an analysis of individual-level BMI, in a very differently scaled and geographically bounded study, on a smaller sample size. The BMI results indicated that even within a small bounded research area spatial heterogeneity may still exist and may provide insights in future analysis (e.g., the community-level racial composition). While relative to the overall analytic units, the bandwidths in the BMI study were large, the local spatial processes continue to play an important role in understanding why some variables (e.g., self-rated health) are more important in certain areas of the map. Our examples strongly provide evidence to support the applicability of GWQR.

Though the value of GWQR has been validated, several issues warrant attention. First, given the relatively large bandwidths found in the BMI analysis and the complexity of GWQR, we currently recommend that this new method may be more reliable on samples of 350 or more observations. GWQR needs to collect enough data points to estimate local models, especially at the extremes of a distribution. This issue deserves further research and drawing on the experience of others with GWR, this topic may be best explored using a simulation-based approach, rather than with empirical, data (Paez 2005; Paez, Farber, and Wheeler 2011). Second, while the literature indicated that heterogeneity is a product of the researchers' perspective (Dutilleul and Legendre 1993; Li and Reynolds 1995), a single indicator that simultaneously tests both statistical and spatial heterogeneity should be developed. Third, the current GWQR approach is limited to a fully non-parametric approach, but we hope to develop a bootstrapping framework to tackle several technical issues raised in the original GWQR paper, such as multicollinearity and statistical testing (Chen et al. 2012; Davison and Hinkley 1997). Finally, while using the five summary statistics and visualizing spatial heterogeneity are appropriate techniques to demonstrate the massive amount of information generated by GWQR, how to fully utilize the results to simultaneously present both statistical and spatial heterogeneity remains a challenge.

In sum, the well-known limitations of the traditional regression approach have sparked the development of GWR and QR. New analytic methods and recent growth in local modeling approaches reinforced the need to focus on issues of heterogeneity (Fotheringham, Brunsdon, and Charlton 2002; Hao and Naiman 2007). In this paper, we have discussed the definitions of statistical and spatial heterogeneity, elaborated on their significance and potential benefits, and offered two empirical examples to support the value and applicability of GWQR in demographic and health research. Demography and health science have a long history of investigating population dynamics and space (Cromley and McLafferty 2011; Goovaerts 2008; Voss 2007; Young and Gotway 2010), and we believe a focus on heterogeneities will help move these fields forward.

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Variables	Minimum	Maximum	Mean	Std. Deviation
Panel A				Deviation
US Mortality Example (N=3,072)				
Age-sex Adjusted Mortality	0.000	18.889	8.908	1.462
(per 1,000 population)				
Deprivation Index	-5.118	21.785	-0.301	2.327
Poverty Rates	0.000	0.524	0.154	0.065
Metropolitan Status	0.000	1.000	0.345	0.475
Panel B				
Philadelphia Obesity Example (N=377)				
Body Mass Index (BMI)	13.705	68.652	30.110	7.191
Self-rated Health (1=excellent, 5=poor)	1.000	5.000	3.077	1.070
Age (continuous)	18.000	90.000	47.698	17.279
Income Level (ordinal)	1.000	12.000	4.769	2.961
Distance to Fruit and Vegetable Store (in miles)	0.063	32.117	2.207	2.096

# Table 1. Descriptive statistics for variables in examples

	Minimum	$Q_1$	Median	Q3	Maximum	S.E. <sup>‡</sup>	Heterogeneous <sup>†</sup>
Q=0.05 (bandwidth=694)							
Intercept	1.780	6.180	6.700	7.290	8.750	0.244	Yes
Deprivation Index	-0.338	-0.043	0.016	0.097	0.467	0.020	Yes
Poverty Rates	-8.430	1.785	5.504	9.008	18.710	1.585	Yes
Metropolitan Status	-1.540	-0.255	-0.048	0.280	2.560	0.127	Yes
<u>Q=0.25 (bandwidth=394)</u>							
Intercept	4.550	6.888	7.527	8.015	9.600	0.131	Yes
Deprivation Index	-0.247	-0.014	0.043	0.103	0.466	0.023	Yes
Poverty Rates	-11.950	2.972	7.048	9.428	28.090	0.741	Yes
Metropolitan Status	-1.050	-0.214	-0.642	0.074	0.850	0.067	Yes
Q=0.50 (bandwidth=301)							
Intercept	4.710	7.384	7.861	8.335	10.401	0.126	Yes
Deprivation Index	-0.240	-0.028	0.027	0.099	0.442	0.024	Yes
Poverty Rates	-9.310	3.967	7.436	10.786	26.000	0.776	Yes
Metropolitan Status	-1.480	-0.234	-0.068	0.064	-0.068	0.053	Yes
<u>Q=0.75 (bandwidth=412)</u>							
Intercept	4.340	7.513	8.211	8.773	10.610	0.125	Yes
Deprivation Index	-0.390	-0.329	0.041	0.114	0.389	0.022	Yes
Poverty Rates	-7.020	5.181	8.668	12.633	28.080	0.731	Yes
Metropolitan Status	-1.430	-0.257	-0.006	0.135	1.050	0.051	Yes
<u>Q=0.95 (bandwidth=630)</u>							
Intercept	4.380	8.198	8.885	9.902	13.240	0.224	Yes
Deprivation Index	-0.404	-0.052	0.018	0.105	0.472	0.042	Yes
Poverty Rates	-7.130	5.769	10.388	13.213	30.340	1.307	Yes
Metropolitan Status	-4.120	-0.433	-0.076	0.129	1.440	0.099	Yes

Table 2. GWQR results for US mortality example at selected quantiles

<sup>†</sup>Heterogeneity is observed if the interquartile range  $(Q_3-Q_1)$  is at least two times greater than the S.E. <sup>‡</sup>Standard errors from the global quantile regression.

Table 5. GwQK fesuits for Phila	Minimum	Q <sub>1</sub>	Median	Q <sub>3</sub>	Maximum	S.E. <sup>‡</sup>	Heterogeneous <sup>†</sup>
Q=0.05 (bandwidth=346)							
Intercept	-0.919	16.378	20.245	22.996	43.006	1.585	Yes
Self-rated Health	-1.955	-0.730	-0.076	0.757	5.362	0.395	Yes
Age	-0.292	-0.009	0.011	0.045	0.130	0.023	Yes
Income Level	-1.407	-0.189	-0.037	0.177	0.414	0.133	Yes
Distance	-2.412	-0.028	0.215	0.714	1.703	0.211	Yes
Q=0.25 (bandwidth=346)							
Intercept	14.273	21.234	22.639	23.785	28.974	1.401	No
Self-rated Health	-0.391	0.535	0.707	0.955	3.028	0.299	No
Age	-0.065	-0.015	0.013	0.028	0.073	0.019	Yes
Income Level	-1.190	-0.176	-0.020	0.092	0.849	0.145	No
Distance	-0.270	-0.061	-0.029	0.042	0.221	0.149	No
<u>Q=0.50 (bandwidth=295)</u>							
Intercept	13.675	21.230	23.930	26.532	33.132	2.733	No
Self-rated Health	-0.740	0.706	1.517	1.800	4.667	0.510	Yes
Age	-0.131	-0.009	0.019	0.041	0.227	0.027	No
Income Level	-0.853	-0.156	-0.061	0.024	0.475	0.182	No
Distance	-0.583	-0.178	-0.038	0.469	2.896	0.272	Yes
<u>Q=0.75 (bandwidth=363)</u>							
Intercept	13.206	23.267	27.006	29.486	37.789	1.808	Yes
Self-rated Health	0.883	1.872	2.485	3.108	5.517	0.445	Yes
Age	-0.108	-0.025	-0.005	0.018	0.068	0.022	No
Income Level	-0.805	-0.163	-0.080	-0.009	0.993	0.177	No
Distance	-1.404	-0.344	-0.137	0.394	2.303	0.274	Yes
<u>Q=0.95 (bandwidth=320)</u>							
Intercept	24.127	33.173	35.025	41.934	51.167	4.586	No
Self-rated Health	-1.826	0.222	1.933	3.075	4.360	1.028	Yes
Age	-0.167	-0.084	-0.040	-0.012	0.128	0.060	No
Income Level	-0.640	0.214	0.589	1.114	2.307	0.576	No
Distance	-2.408	-0.592	-0.170	0.623	3.445	1.217	No

Table 3. GWQR results for Philadelphia obesity sample at selected quantiles

†Heterogeneity is observed if the interquartile range  $(Q_3-Q_1)$  is at least two times greater than the S.E. ‡Standard errors from the global quantile regression.



Figure 1. Heterogeneous associations between the deprivation index and mortality by quantiles



Figure 2. Heterogeneous associations between self-rated health and BMI by quantiles