Old-age support and demographic change in developing countries. A cultural transmission model

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Javier Olivera*

Department of Economics, Catholic University of Leuven

Abstract: We model intergenerational old-age support within the context of a developing country that faces demographic changes: declining fertility and increasing life expectancy. We attempt to answer if agents will be able to support their parents during the next generations and under what conditions. For this purpose we use a three period overlapping generations model and a cultural transmission process, in which agents may be socialized to different cultural family models (old-age supporters and non-supporters). As life expectancy increases, we find conditions under which a reduced fertility rate is compatible with the expectation to be supported during old-age. This offers an additional explanation for persistency of family old-age support in developing countries with demographic change.

*E-mail address: javier.olivera@econ.kuleuven.be Fax: ++3216326796.
Address: Naamsestraat 69, Leuven. 3000. Belgium.

1. Introduction

The most notorious stylised facts of the demographic change are the continued fall of the fertility rate and infant mortality and the increase of the life expectancy, which lately reduce the population growth and increase the proportion of the elderly. Although at different pace, almost all societies are experiencing the demographic transition. This is well advanced and close to complete in developed countries and incipient or sustained in developing countries.

According to the theory of intergenerational wealth flows (Caldwell, 2005) or the old-age security motive (Nugent, 1985), the fall of fertility in developing countries is a result of the decline of the old-age insurance value of children. In contrast, for the economic theory of the family (Becker and Barro, 1988; Willis, 1973, 1982), the driver of the fertility decline is the change in the available economic opportunities (e.g. high rearing costs, more female labour participation; more education). Some economists have also acknowledged the importance of culture and changing preferences for the determination of fertility (see Pollack and Watkins, 1993), and cross-cultural psychologists have provided appealing empirical support. For instance, Kagitcibasi (1982; 2007) and Trommsdorff (2009)[1] argue that the economic value of children has declined (and will continue decreasing) in favour of a more prominent role for an emotional or psychological value of children, which will lead to the reduction of fertility as well as a change in how children and old-age care are valued in society. This perspective is interesting because it takes in account culture as an important driver of the evolution of fertility and old-age support. Furthermore, Fernandez and Fogli (2006) show econometrically that both culture (measured through fertility rate of the country of ancestry) and personal experience matters on fertility decisions. Although culture is also acknowledged in Blackburn and Cipriani (2005) as an important factor to explain the decline of fertility, this is not explored. Instead, the authors give a leading role to the intergenerational transfers to explain the demographic transition. Their model ends up showing different equilibria which are a close representation of what we observe in the reality: economies with high levels of development, low fertility and strong downstream intergenerational transfers and economies with low development, high fertility and strong upstream intergenerational transfers. In this chapter, we consider that tensions between

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[1] This thesis is built on the basis of the results of the Value of Children study implemented during the 70's and replicated during the 2000's in a pool of societies that differ in culture and degree of modernization.

different cultural values towards old-age support within a society will play a role in the determination of the fertility rate.

If we accept that parents have children to insure against the old-age risk, the continued decrease of fertility rates and increase of life expectancy imply that children must support their old parents longer and with the help of fewer siblings, which increase the cost of this support. In developed countries social security institutions help to overcome this burden; but in developing countries these institutions are scarce, the family being the main mean for insuring against old-age. Therefore, the demographic change poses a challenge for these countries. Given that the family old-age insurance is well spread in developing countries, what is the rationale behind the long run demographic projections that show parents living longer and having fewer children? One may think of a more intensive use of social security as a way to cope with this challenge, but this might not be the case. For instance, some Latin American countries launched pension systems in order to alleviate the old-age risk, but there are still a considerable proportion of families that do not participate in such schemes[2]. How can we reconcile the decision of individuals of having fewer children and expecting family old-age support within societies where more institutionalised forms of old-age support are scarce and longevity is increasing?

The goal of this chapter is twofold. First, as we consider that in developing countries culture matters on the decision of fertility and old-age support, we build a framework where socialization to certain cultural values plays a central role in the old-age support and fertility. This model can describe the demographic change in developing countries in presence of important norms of family old-age support. Second, we are interested to highlight the consequences of the demographic change on the ability of the households to adequately insure their older members. For these aims, we propose an OLG model of three periods with a cultural transmission process *à la* Bisin and Verdier (2001). In this process parents transmit their preferences or traits to their children, who must choose one of those traits under the influence of vertical (parental) and oblique (societal) cultural socialization. This transmission is made via a costly socialization effort. We consider that parents inculcate some cultural values related to the importance of old-age support (e.g. filial piety) to their

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[2] These countries favoured pension systems based on individual capital accounts in a major reform wave during the 90's, but the number of enrolled and contributors in the pension system have decreased instead of increase (Arenas de Mesa and Mesa-Lago, 2006). Hence, the family still plays the most important role for old-age protection. Furthermore, those countries will experience a significant drop of the TFR and increase of the life expectancy. Between 1955 and 2050, the TFR will drop from 5.89 to 1.85, and the life expectancy at birth will rise from 59.8 to 79.6 (database of the Economic Commission for Latin American and the Caribbean).

children with the goal to obtain economic support during old-age. We denote this cultural trait "old-age supporters". At the same time, children are exposed to other cultural values that could imply a different response to old-age care. For example, the users of social security or any other way of own old-age saving may deliver the cultural value of being independent, i.e. neither supporting old-age parents nor being supported by own children. This trait is called "non old-age supporters". That fact that there are only two cultural traits is evidently a reduction of the reality, but is enough to reflect the tension between different visions on the value of children. This tension is acknowledged in Kagitcibasi (1982, 2007) when the author describes the evolution of the value of children as a result of the development and urbanization process.

There are other applications of the cultural transmission model (see the survey of Bisin and Verdier, 2010). Baudin (2010) uses a cultural transmission framework to study the population dynamics of a society where two cultural types with different fertility norms (modernists and traditionalists) compete. The work of Bisin and Verdier (2001) itself also discusses the population dynamics of different cultural transmission technologies. To our knowledge, the study of intergenerational transfers has not yet been studied with this framework. In contrast, there are some works dealing with intergenerational transfers within endogenous fertility growth models but the culture component is missing. For instance, Raut and Srinivasan (1994), Chakrabarti (1999) and Morand (1999) show how the economies arrive to different equilibria of fertility, growth and human capital when intergenerational transfers are considered. In an OLG model, Palivos (2001) shows how different family–size norms lead to multiple equilibria, but intergenerational transfers are not considered. We contribute to the literature by studying intergenerational support in developing countries with an OLG model where fertility, socialization effort and the distribution of cultural types are endogenous. In addition, the results of the model are stressed by considering exogenous improvements in longevity.

One of the major results of this chapter is to account for the demographic change by showing how the increase of longevity may reduce fertility and the expected old-age transfer from children. The mechanism is that the old-age supporter type receives an expected transfer amount from her children that is influenced by the direct socialization of the parent (directed to her own type) and the proportion of types in society. The socialization effort affects directly the probability of embracing the parental type, so that the parent must increase their cultural socialization effort to instil enough old-age transfers from her children. As socialization effort is costly, fewer resources are devoted to childbearing

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and hence the fertility of this type drops. The non old-age supporter types react by increasing their socialization effort because they want to offset the influence of the other type in the cultural transmission to their own children. Therefore, they reallocate resources from childrearing to socialization, and this pushes down their fertility. Overall, fertility of both types declines and socialization effort increases. While we analyse the long run equilibria, we conjecture that a continued increase of longevity and the cultural response of the old-age supporter can jeopardise the existence of this trait. This is a well cast example of non efficient cultural reaction to the economic environment. Moreover, we inspect the effects of implementing a compulsory pension scheme and a more tolerant setting between the types on our conjecture. We find that these "solutions" help to improve the ability of children to support parents and avoid or delay the extinction of the old-age supporter trait.

The rest of the chapter is organized as follows. The model is set out in section 2. Section 3 deals with the effects of the demographic transition and section 4 shows two extensions of the model. Finally, section 5 concludes.

2. The model

We consider an overlapping generations model with three periods: childhood, adulthood and old age. During childhood, individuals are socialized towards certain cultural models by their parents (vertical socialization) and society (oblique socialization) following a cultural transmission process *à la* Bisin and Verdier (2001). Individuals are inculcated family cultural values on the commitment of giving old-age support to parents. We restrict the set of cultural types to two: 1) old-age supporters and 2) non old-age-supporters. A type 1 person supports financially her old parents and expects to be supported by her children in old-age as well. A type 2 person does not support her parents, saves for her own consumption in old-age, and does not expect to be supported by her children.

q_t represents the fraction of individuals with cultural type 1 and $1 - q_t$ is the fraction of individuals with type 2. Subscript t indicates time. Children are born without any cultural trait and are first exposed to the parent's cultural type. Socialization to parental cultural type i occurs with probability d_t^i, $i \in \{1,2\}$. If the socialization of the child towards her parent's cultural type i fails (which occurs with probability $1 - d_t^i$), she picks another cultural type randomly from the adult population, which means that type 1 is chosen with probability q_t and type 2 with probability $1 - q_t$. The probability that the child of a parent with cultural type *i* is socialized to cultural type *j* is P_t^{ij}. The transition probabilities for *i,j* $\in \{1,2\}$ are:

$$P_t^{11} = d_t^1 + (1 - d_t^1)q_t \quad \text{and} \quad P_t^{12} = (1 - d_t^1)(1 - q_t) \qquad (1)$$

$$P_t^{22} = d_t^2 + (1 - d_t^2)(1 - q_t) \quad \text{and} \quad P_t^{21} = (1 - d_t^2)q_t \qquad (2)$$

The success of vertical socialization depends on the effort τ_t^i of the parent to socialize her child to her own cultural type, i.e. $d_t^i = d(\tau_t^i)$. This effort costs $c(\tau_t^i)$. Depending on how the cultural transmission technology is specified, vertical and oblique cultural transmission may be substitutes or complements. In the first case, the parent belonging to the major type has less incentives to socialize her children to her cultural type because she can free-ride from her dominant position in society[3]. If more parents in society have type i, the probability of having a child with the same type i is larger; however, the less dominant types will increase their vertical socialization effort in order to not disappear. In the second case, parents from the minority type socialize their children to the major type. An example of this is cultural assimilation. The conditions and set up of the cultural transmission model will lead to the specific socialization technology.

The parent is able to know the consequences of the possible socio-economic actions of her children but she evaluates those actions through her own preferences. It is assumed that the parents always prefer to have children with the same cultural type. In this sense, the parent acts myopically or paternalistically towards her children. We denote $\theta^{ij} \, \forall i, j \in \{1,2\}$ as the increase in the utility of a type i parent who has a child with type j. After normalization, we assume:

Assumption 1: for $i, j \in \{1,2\}$, with $i \neq j$, $\theta^{ii} > \theta^{ij}$ and

$$\begin{bmatrix} \theta^{11} & \theta^{12} \\ \theta^{21} & \theta^{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (3)$$

Having a child with the same cultural type is the best state for a parent $\theta^{ii} = 1 \, \forall i$, but a child embracing another type is the least tolerated state ($\theta^{ij} = 0, \forall i \neq j$) whatever the type of the parent. The difference $\Delta\theta^i = \theta^{ii} - \theta^{ij}$ measures the intolerance of an individual with cultural type i towards type j; therefore in our matrix, both cultural types disfavour the other types symmetrically (Bisin et al, 2009). However, it is arguable that a type 2 parent may be

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[3] Bisin and Verdier (2001) refer to this as the cultural substitution property, which is needed to reach a distributional equilibrium with heterogeneous types.

more tolerant to cultural deviation than a type 1 parent. We study this possibility in a further section.

Assumption 2: a) $d_t^i = \tau_t^i \in [0,1]$; b) $c(\tau_t^i) = c\tau_t^i$, $c > 0$.

We assume that the probability of success of vertical socialization equals the socialization effort made by the parent. Although not evident here, we will show later that socialization effort is an endogenous response from parents that depends on q_t as well. The cost of one unit of socialization effort is *c*, which is expressed as time cost and is common to all cultural types.

Once the individual has embraced a cultural type at the end of childhood, she has to decide her consumption level, fertility and socialization effort in her adulthood. This individual is endowed with one unit of time which she must allocate to labour, childrearing, old-age financial support for her parents (or savings) and effort to socialize her children towards her own cultural type. Type 1 individuals decide their socialization effort, fertility and consumption respecting the following budget constraints:

$$C_t^1 = w_t(1 - n_t^1(\rho + c\tau_t^1) - b_t) \geq v \tag{4}$$

$$C_{t+1}^1 = w_{t+1}n_t^1 b_{t+1}P_t^{11} \geq m \tag{5}$$

The adult receives wage w_t per unit of time worked. We do not distinguish wage by cultural type and take it as given (similar to Chakrabarti, 1999 and Palivos, 2001). It is also considered that preferences on consumption are defined above subsistence levels of consumption in adulthood (*v*) and old-age (*m*). Adult consumption C_t^1 is financed with the wage net of the time cost ρ of rearing children, the time cost of socialization effort and the fraction b_t transferred to the old parent. This last term corresponds to the share of a total old-age transfer $B \in (0,1)$ that each child should transfer if all siblings were type 1, i.e. $b_t = B/n_{t-1}^1$. As considered by some OLG growth models with old-age support (e.g. Raut and Srinivasan, 1994; Chakrabarti, 1999 and Morand, 1999), the old-age financial support is an exogenous fraction of the labour time. Children who adopt type 1 values do not transfer money on top of the share resulting from equal division among siblings; which means that parents are not financially compensated for possible cultural deviation. We can think of *B* as a result of a social norm under which children must support (equally) old parents with a transfer. Given that cultural deviation is possible, the value of *B* is just a reference and it is

assumed that it is well above subsistence level. For the same reason, consumption in old-age C_{t+1}^1 is an expected value[4]. For now, we assume that minimum consumption constraints are not binding, but later on we relax this assumption.

Type 2 adults choose their socialization effort, fertility, consumption and savings according to the following budget constraints:

$$C_t^2 = w_t(1 - n_t^2(\rho + c\tau_t^2) - s_t) \geq v \qquad (6)$$

$$C_{t+1}^2 = w_t s_t R \geq m \qquad (7)$$

Type 2 does not support financially her parent; instead, she saves a fraction s_t during adulthood and consumes all her savings (times the interest factor $R \geq 1$) during old-age. In case of cultural deviation, we rule out the possibility that type 1 children donate a transfer to a type 2 parent. As specified before, b_t is only intended for parents who are type 1. To give support to this assumption, we may invoke a sort of demonstration effect within three generations such as argued in Cox and Stark (2005). This effect is observed when a parent supplies support to her own parent to instil the same behaviour from her children. Thus, we may consider that a type 1 child will not support her type 2 parent as this last one did not support her own parent. In addition to the budget constraints, it is necessary to add some time and non negativity constraints:

$$1 - (\rho + c\tau_t^1)n_t^1 - b_t \in (0,1) \qquad (8)$$

$$1 - (\rho + c\tau_t^2)n_t^2 - s_t \in (0,1) \qquad (9)$$

$$n_t^i \geq 0; \tau_t^i \geq 0; s_t \geq 0 \qquad (10)$$

All adults maximize the following utility function. We use a logarithmic function for tractability reasons.

$$U_t^i = ln(C_t^i) + \delta ln(C_{t+1}^i) + \beta ln(n_t^i) + ln\left(n_t^i(\theta^{ii}P_t^{ii} + \theta^{ij}P_t^{ij})\right) \qquad (11)$$

As in other OLG models (Raut and Srinivasan, 1994; Morand, 1999; Palivos, 2001) the only productive stage is adulthood and the individuals do not consume during childhood,

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[4] Note that type 1 adults use probabilities of period t to construct the expected transfer corresponding to $t+1$, due to their inability of predicting future probabilities. Furthermore, given that $b_{t+1} = B/n_t^1$, equation 5 can be reduced to $w_{t+1}BP_t^{11}$.

there are no bequests and both parents (mother and father) are only one entity. A type i derives utility from her consumption during adulthood and old-age. The parameter δ is a time discount factor. The number of children n_t^i also generates satisfaction to adults through the parameter β. Endogenous fertility models as in Becker and Barro (1988) and Dahan and Tsiddon (1998) also include the number of children as a source of utility for the parent. The last term in equation 11 represents the gains in utility that an adult expects from the cultural transmission process; as mentioned before, a parent prefers a child with the same cultural type. Moreover, the restrictions on the parameters are $\delta, \beta, \rho \in (0,1)$ and one has to assume[5] $\beta > \delta$ in order to get a positive number of children at any $q_t \in (0,1)$. Finally, the dynamics of the model is governed by:

$$q_{t+1} = \frac{q_t + q_t n_t^1 P_t^{11} + (1-q_t) n_t^2 P_t^{21}}{1 + q_t n_t^1 + (1-q_t) n_t^2} \tag{12}$$

Equation 12 shows the dynamics of distribution of types through the proportion of cultural type 1 in period t+1. If the total population in period t is N_t , then $q_t N_t$ are type 1 and $(1 - q_t)N_t$ are type 2. The number of type 1 individuals at t+1 is formed by i) the children of type 1 parents who becomes type 1: $q_t N_t n_t^1 P_t^{11}$; ii) the children of type 2 parents who become type 1: $(1 - q_t)N_t n_t^2 P_t^{21}$ and iii) the type 1 parents who become old parents: $q_t N_t$. If we divide the sum of all these individuals over the total population at t+1: $q_t N_t n_t^1 + (1 - q_t)N_t n_t^2 + N_t$ and cancel the term N_t, we obtain equation 12.

2.1 Behaviour of cultural type 1

Under assumptions 1-2, equations 1-2 and provided that subsistence consumption levels are not binding, each adult finds the optimal values of her consumption, fertility and socialization effort $(C_t^1 ; C_{t+1}^1; n_t^1; \tau_t^1)$ by maximizing equation 11 subject to equations 4 and 5. As ρ is the share of time dedicated to childbearing and the social norm transfer $b_t = B/n_{t-1}^1$ cannot be dishonoured by type 1 individuals, the maximum number of children is limited to $\frac{(1-b_t)}{\rho}$.

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[5] Furthermore, this assumption leads us to get the expected signs for partial derivatives ($\partial n_t^i/\partial \rho < 0$ and $\partial \tau_t^i/\partial c < 0$).

$$n_t^1 = \begin{cases} \frac{(1-b_t)(\beta-\delta)(1-q_t)}{(2+\beta)(\rho-q_t(\rho+c))} & if\ q_t \le x_n \\ \frac{(1-b_t)}{\rho} & if\ q_t > x_n \end{cases} \quad (13) \qquad \tau_t^1 = \begin{cases} \frac{(1-q_t)(1+\delta)\rho-cq_t(1+\beta)}{c(\beta-\delta)(1-q_t)} & if\ q_t \le x_\tau \\ 0 & if\ q_t > x_\tau \end{cases} \quad (14)$$

where $x_n = \frac{\rho(2+\delta)}{c(2+\beta)+\rho(2+\delta)} > x_\tau = \frac{\rho(1+\delta)}{c(1+\beta)+\rho(1+\delta)}$. For a given period t, figure 1 shows the possible interior solutions of n_t^1 and τ_t^1 as a function of the distribution of types in the same period.

FIGURE 1: FERTILITY AND SOCIALIZATION EFFORT FOR TYPE 1
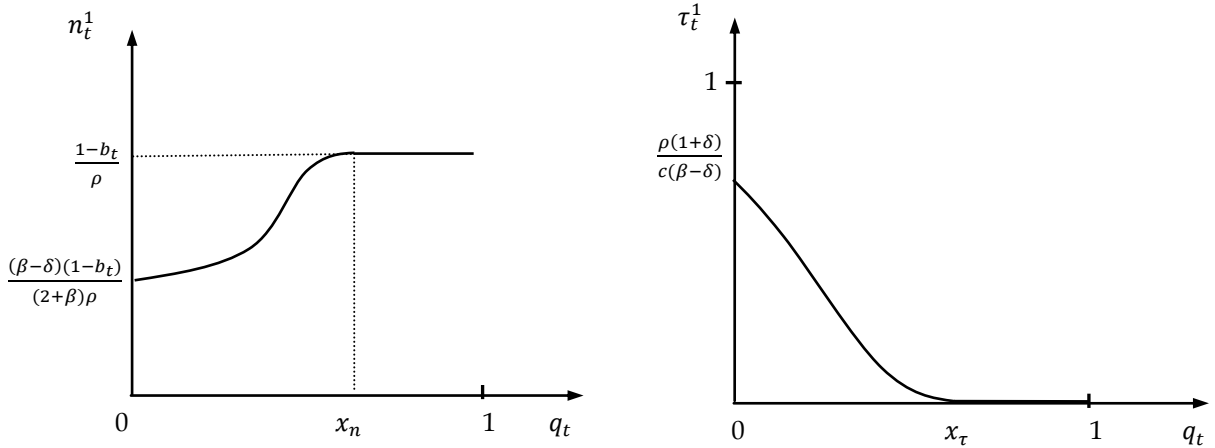
Note that when not constrained, $\frac{\partial n_t^1}{\partial q_t} > 0$ and $\frac{\partial \tau_t^1}{\partial q_t} < 0$. The kink in the socialization effort comes from the non negativity constraint on τ_t^1 . The probability of having a child with cultural type 1 rises with the share of this type in the adult population, therefore the individual chooses a higher fertility rate in order to increase the gains in utility of having a child with the same cultural type. The result $\frac{\partial \tau_t^1}{\partial q_t} < 0$ is the cultural substitution property invoked in Bisin and Verdier (2001). This means that a type 1 individual expends less time in socialization and relies more on the oblique socialization by the society to socialize her children. In this way, the majority group invest less in socialization. Under this property, the authors prove that an equilibrium with a heterogeneous distribution of types is possible.

The minimum number of children is $\frac{(\beta-\delta)(1-b_t)}{(2+\beta)\rho}$. It is interesting to note that fertility depends negatively on the old-age transfer, i.e. if the parent lives longer and thus needs more money in old-age, then fertility declines. We explore this in more detail in section 3. The maximum value that optimal socialization effort may reach is $\frac{\rho(1+\delta)}{c(\beta-\delta)}$ or unity.

2.2 Behaviour of cultural type 2

Similar to type 1, type 2 finds the optimal values of her consumption, fertility and socialization effort $(C_t^2 ; C_{t+1}^2; n_t^2; \tau_t^2; s_t)$ by maximizing equation 11 subject to equations 6 and 7. As before, the highest rate of fertility is bounded by the share of time dedicated to childbearing and the savings needed to consume in old-age: $\frac{(1-s_t)}{\rho}$.
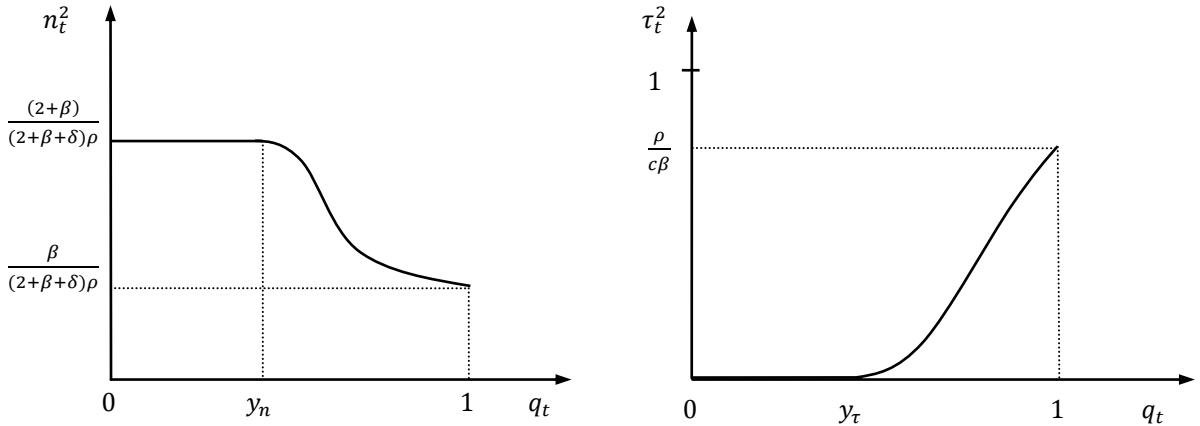
$$n_t^2 = \begin{cases} \frac{q_t\beta}{(2+\delta+\beta)(q_t(\rho+c)-c)} & if \; q_t > y_n \\ \frac{2+\beta}{(2+\delta+\beta)\rho} & if \; q_t \leq y_n \end{cases} \quad (15) \qquad \tau_t^2 = \begin{cases} \frac{q_t\rho-c(1+\beta)(1-q_t)}{cq_t\beta} & if \; q_t > y_\tau \\ 0 & if \; q_t \leq y_\tau \end{cases} \quad (16)$$

$$s_t = \frac{\delta}{2+\delta+\beta} \quad (17)$$

where $y_n = \frac{c(2+\beta)}{c(2+\beta)+2\rho} < y_\tau = \frac{c(1+\beta)}{c(1+\beta)+\rho}$. To ease comparisons, figure 2 shows the possible interior solutions, for a given period t, of n_t^2 and τ_t^2 as a function of type 1's distribution.

FIGURE 2: FERTILITY AND SOCIALIZATION EFFORT FOR TYPE 2

We observe $\frac{\partial n_t^2}{\partial q_t} < 0$ and $\frac{\partial \tau_t^2}{\partial q_t} > 0$ because of the same reasons explained for type 1; the latter derivative confirms that vertical and oblique cultural transmissions are substitutes. The number of children is between $\frac{\beta}{(2+\beta+\delta)\rho}$ and $\frac{2+\beta}{(2+\beta+\delta)\rho}$. Socialization effort is zero when $q_t < y_\tau$ due to the non negativity constraint on τ_t^2 and it cannot be larger than $\frac{\rho}{c\beta}$ or unity. The savings rate depends on the altruism and discount parameters rather than on the interest factor due to the logarithmic form of utility and the fact that constraints are not binding. This result is similar to that of De la Croix and Doepke (2003).

2.3 Dynamics

The dynamics of the distribution of types is governed by $\Delta q_t = q_{t+1} - q_t$. After some algebra applied on equation 12, we can arrive to the expression $\Delta q_t = \frac{q_t(1-q_t)(n_t^1\tau_t^1 - n_t^2\tau_t^2)}{1+n_t^2+q_t(n_t^1-n_t^2)}$. It is easily observable that $\{0;1\}$ are equilibria of the system as well as all q_t which solve $n_t^1\tau_t^1 - n_t^2\tau_t^2 = 0$. The stability of these equilibria is not easy to assess given that the system of equation 12 is non-linear of second order and has kinks. However, it is possible to find that $\{0;1\}$ are unstable, while the other equilibria arise from chaotic dynamics, so that it is hard to study their stability. Nonetheless, we can show that the system has an analytical solution until critical value $c=c^*$; once this threshold is passed (i.e. when $c < c^*$), the system undergoes a bifurcation and then presents chaotic dynamics[6]. Figure 3 shows the optimal dynamic paths for each value of c (at step of 0.0025) considered in the computation of the path[7].

FIGURE 3: POPULATION DYNAMICS
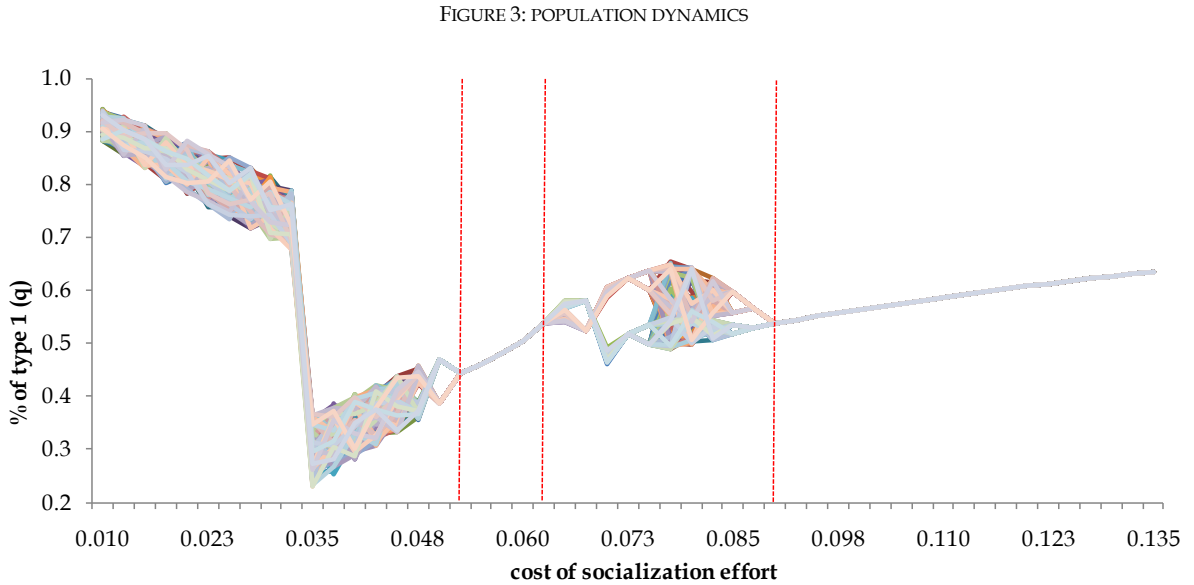
Starting from the right we observe that there is only one steady state (hyperbolic fixed point in bifurcation theory), which decreases with the reduction of c; and then, the bifurcation occurs around $c=0.0897$. After this threshold, the system undergoes a period-doubling bifurcation and then a chaotic dynamics. Interestingly, the system reaches another set of hyperbolic fixed points at $c \in [0.055, 0.063]$. The threshold c^* comes from $x_\tau = y_\tau$, i.e

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[6] Given that it is possible to find real life or standard values used in literature for all parameters, but c, we choose this parameter to evaluate the dynamics of the model.
[7] For figure 3, we use $\rho=0.154$; $\delta=0.3433$; $\beta=0.99$; $q_0=0.95$. The same chaotic shape is found with the use of other parameter values and initial conditions. Further, in section 3.3 we will argue about the values of these parameters.

$c^* = \frac{\rho\sqrt{1+\delta}}{1+\beta}$. Thus, if $c < c^*$ (or equivalently $x_\tau > y_\tau$), the system undergoes a bifurcation and then chaos.

Proposition 1: For initial conditions $(q_0; n_0^i; \tau_0^i)$, with no binding consumption constraints and assumptions 1-2, $\{0; 1; q^* | n_t^1(q^*)\tau_t^1(q^*) - n_t^2(q^*)\tau_t^2(q^*) = 0\}$ are equilibria of the dynamic system 1-17. $\{0;1\}$ are unstable, and q^* is stable if $x_\tau \leq y_\tau$. The stability of q^* is only locally guaranteed if $x_\tau > y_\tau$ because the dynamic system undergoes a bifurcation when $c < c^* = \frac{\rho\sqrt{1+\delta}}{1+\beta}$.

Proof: see appendix.

The point q^* means that the system can reach an heterogeneous distribution of types, which is stable if $x_\tau \leq y_\tau$. In order to ease the interpretation of the dynamics hereafter, we will assume the following restrictions on the parameters: $x_\tau \leq y_\tau$. It is straightforward to obtain $q^* \in [x_\tau, y_\tau]$ as τ_t^1 and τ_t^2 are equal to zero for $q_t \geq x_\tau$ and $q_t \leq y_\tau$, respectively. Under these dynamics the socialization effort of both types is zero at equilibrium[8], which is obviously a particular case of the more general Proposition 1 (which allows for non-zero values of socialization efforts at steady states). However, this is sufficient to easily describe how types interplay with each other given their optimal responses on fertility and socialization. Recall that for values $x_\tau > y_\tau$ there are multiple equilibria with not assured stability, so that we would not be able to draw regular relations between types. Figure 4 represents the dynamics when $x_\tau \leq y_\tau$.

FIGURE 4: DYNAMICS OF THE DISTRIBUTION OF TYPES

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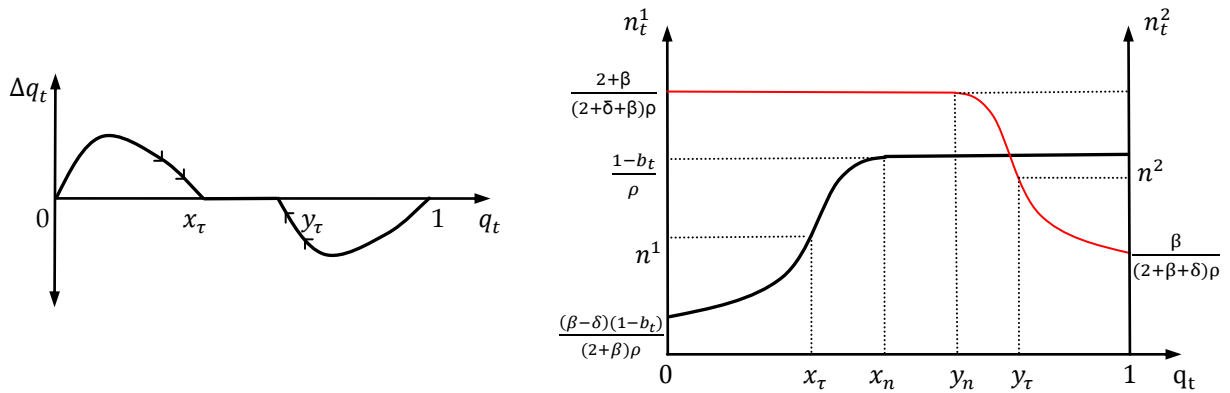
[8] This dynamics may be observed in societies where socialization effort is expensive.

The initial conditions for the distributions of types (q_0) might be placed at any point in [0,1]. In the interval (x_τ, y_τ) adults do not make any socialization effort and fertility is determined without any influence of the distribution of types. It means that the initial distribution of types satisfies every individual. The most interesting cases arrive when $q_0 \in (0, x_\tau]$ or $q_0 \in [y_\tau, 1)$. In the first case, the proportion of type 1 is low in society, so that type 1's children have a high probability of embracing type 2; this, in turn, reduces the expected value of the old-age support. If individuals with type 1 want to be supported enough during old-age by their children, they have two means: socialize more intensively their children towards their own type and having more children. As a result, the proportion of type 1 increases (left panel of figure 4). Given the substitutability between vertical and oblique socialization, τ_t^1 declines over time and stops at $q_t = x_\tau$ (right panel of figure 1). The other reaction of type 1 is to increase fertility over time up to n^1 (right panel of figure 4). Thus, at point x_τ individuals with type 1 reach a desirable distribution of types.

If $q_0 \in [y_\tau, 1)$, type 2's children have a high probability of embracing the other cultural type. Given that a type 2 adult prefers a child with her own cultural type, she increases her socialization effort and fertility in order to obtain a desirable distribution of types such that she can derive enough utility from the cultural alignment between her and her child. In this process, the share of type 2 increases as observed in the left panel of figure 4. Socialization effort decreases and fertility increases over time (figure 2 and right panel of figure 4, respectively) up to $q_t = y_\tau$, with $\tau_t^2 = 0$ and n^2 being the final values of socialization and fertility.

3 Demographic changes

3.1 Living longer

As mentioned in the introduction, the continued increase of longevity is well-documented in many and different populations. This phenomenon may affect decisions on fertility and other relevant variables, and importantly, it might challenge the financial support from children to old parents in populations where old-age family insurance is prominent. We can use our theoretical framework to highlight the possible consequences of higher longevity, which we treat as exogenous[9].

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[9] Although, there are models dealing with the endogenization of life expectancy (for instance Ponthiere (2010) relates longevity as an outcome of lifestyle and Leroux et al (2011) include genetic

A way to easily introduce the rise of life expectancy in the model is to assume that an increase of life expectancy is equivalent to an increase of the minimum old-age consumption, m. We therefore consider the case that constraint 5 is binding. We also assume that adulthood consumption constraint is not binding. Indeed, we will keep this assumption through the rest of this chapter given that we focus on the effects of the increase of life expectancy and such constraint does not change our model's main implications. As the inclusion of a binding old-age constraint produces optimal responses that depend on wage levels, we assume no growth of wages ($w_t = w_{t+1} = w$) in order to simplify the analysis, otherwise we should add human capital accumulation which is beyond the focus of our model. Equations 18 and 19 show the optimal fertility and socialization effort when m binds for type 1:

$$
\hat{n}_t^1 = \begin{cases} \dfrac{Bw(1+\beta)(1-b_t)(1-q_t)}{(2+\beta)\left(cm+Bw(\rho-q_t(\rho+c))\right)} & if \ q_t \leq \hat{x}_n \\ \\ \dfrac{(1-b_t)}{\rho} & if \ q_t > \hat{x}_n \end{cases} \quad (18) \qquad \hat{\tau}_t^1 = \begin{cases} \dfrac{m-Bwq_t}{Bw(1-q_t)} & if \ q_t \leq \hat{x}_\tau \\ 0 & if \ q_t > \hat{x}_\tau \end{cases} \quad (19)
$$

where $\hat{x}_n = \dfrac{cm(2+\beta)+Bw\rho}{Bw(2c+\rho+c\beta)} > \hat{x}_\tau = \dfrac{m}{Bw}$.

Results of equation 18 and 19 are possible only if $\frac{m}{Bw} < 1$; otherwise the constraint cannot be satisfied[10]. Different from the previous program, fertility and socialization effort are affected by wages and minimum old-age consumption level m. As evident from equations 18 and 19, a rise of life expectancy in period t reduces fertility and increases socialization effort. An increase of m triggers a positive variation of socialization effort from parents in order to raise their chances to receive enough transfers to fulfil minimum old-age consumption, which implies that there are fewer resources to invest in childbearing and hence fertility falls. From the re-arranged equation 5, $C_{t+1}^1 = wB(q_t + (1 - q_t)\hat{\tau}_t^1) = m$, it is clear that a parent must increase her socialization effort to cope with a rise of life expectancy in period t. Certainly, an increase of her fertility in period t could only improve the expected value of old-age consumption for the next generation of parents and not for her. The reason

and behavioural factors to explain differences in longevities), we treat this variable as exogenous to avoid unnecessary complexity in the model.

[10] Equation 19 may be rearranged as $\hat{\tau}_t^1 = \frac{1}{1-q_t}\left(\frac{m}{Bw} - q_t\right)$, which should be greater than unity if $\frac{m}{Bw} \geq 1$. Thus, τ_t^1 is truncated to 1. Replacing this value into equation 5 leads to $C_{t+1}^1 = wB \leq m$, i.e. the transfers from children might be insufficient to cover minimum old-age consumption.

is that a rise of n_t^1 in t would increase the proportion of type 1 in $t+1$ ($\frac{\partial q_{t+1}}{\partial n_t^1} > 0$ from equation 12) and hence the corresponding transition probability that affects the expected transfer amount (P_{t+1}^{11}). Therefore, the adult in period t does not find it useful to increase her fertility to meet the value of m in old-age. Indeed, she reduces her fertility to free resources and reallocate them to socialization effort.

The old-age transfer B is considered fixed and exogenous, arising from a social norm related to support for elderly parents. However, it is plausible that a rise of life expectancy may eventually push up this value. Comparative statistics of a change in the transfer B might illustrate the effects on fertility and socialization. As expected, $\frac{\partial \hat{\tau}_t^1}{\partial B} < 0$ from equation 19, i.e. parents need to socialize their children less intensively due to the higher transfer B. The effect of the transfer B on fertility is less clear and depends on the position of q_t and on the value of B when m begins to bind. Proposition 2 states these effects for type 1:

Proposition 2: a) if m binds at any $\hat{q}_t < x_\tau$, then i) $\frac{\partial \hat{n}_t^1}{\partial B} > 0$ if $B < z$ and ii) $\frac{\partial \hat{n}_t^1}{\partial B} \leq 0$ if $B \geq z$; with threshold $z = \frac{n_{t-1}w(1+\delta)-m(\beta-\delta)}{2w(1+\delta)}$. b) if m binds at any $\hat{q}_t \geq x_\tau$, then $\frac{\partial \hat{n}_t^1}{\partial B} < 0$.

Proof: see appendix.

A rise of the old-age transfer will increase fertility only if the original value of B is small enough (less than threshold z). As we pointed out before, a higher value of B reduces $\hat{\tau}_t^1$ which in turn reduces utility (last term of equation 11). However, this cultural loss might be compensated by the adult with an increase of her fertility. Of course, the increase of B and fertility also pushes down utility through the reduction of the first period consumption as the adult must raise the transfer given to her own parent and pay for more children. But these extra expenses can be financed with the resources freed up from socialization effort. In the case that B is large, the burden of paying the old-age transfer is already heavy, so that an increase of fertility would only deteriorate first period consumption without compensating the cultural loss. Therefore, reducing fertility is the best response from the adult when B is already large enough.

If old-age consumption constraints are binding for type 2 ($C_{t+1}^2 = sw_t R \leq m$), the optimal responses are:

$$\hat{n}_t^2 = \begin{cases} \dfrac{q_t\beta(Rw-m)}{Rw(2+\beta)(q_t(\rho+c)-c)} & if\ q_t \geq y_n \\[2mm] \dfrac{Rw-m}{Rw\rho} & if\ q_t < y_n \end{cases} \quad (20) \qquad \hat{\tau}_t^2 = \begin{cases} \dfrac{q_t-c(1+\beta)(1-q_t)}{cq_t\beta} & if\ q_t \geq y_\tau \\[2mm] 0 & if\ q_t < y_\tau \end{cases} \quad (21)$$

$$\hat{s}_t = \frac{m}{wR} \qquad (22)$$

As in the program without binding constraints, $\frac{\partial \hat{n}_t^2}{\partial q_t} < 0$ and $\frac{\partial \hat{\tau}_t^2}{\partial q_t} > 0$. This time fertility is positively affected by the interest factor and by wages due to income effects. A rise of R (or w) lowers the savings rate needed to reach minimum old-age consumption and frees resources that may be invested in childbearing. Different from the program without binding constraints, the savings rate is determined by the interest factor in order to get exactly m during old-age. A rise of m in period t reduces fertility because the adult must meet the higher value of m by devoting more resources to savings and less to childrearing. Furthermore, the rise of m does not affect socialization effort in period t, which is explained by the fact that type 2 adults do not need to socialize their children to their cultural type to reach the old-age consumption m[11]. They socialize children solely for cultural reasons.

3.2 Distribution of types

As we are interested in the evolution of old-age support in developing countries, we will focus on societies with initial conditions $q_0 \approx 1$. These societies are mainly populated by type 1 individuals, the economic value of children is high and the social norm of supporting parents is extended. However, these characteristics and values will start to change at some point of the society's development which may lead to the formation of other values incompatible with old-age family support. This evolution is a consequence of (and reinforced by) forces such as urbanization, social security, modernization. We hypothesise that cultural transmission models are valid mechanisms for such change of values[12].

If $q_0 \approx 1$, the final equilibrium distribution of types ends at $y_\tau = \frac{c(1+\beta)}{c(1+\beta)+\rho}$. It is straightforward to observe how a change of the value of parameters may alter this equilibrium[13]. This would be the final equilibrium if life expectancy -that is equivalent to minimum old-age consumption- did not play a role. The top panel of Figure 5 plots the old-age consumption level of type 1 for each distribution. Given that $C_{t+1}^1 = \frac{wB(1+\delta)[\rho-q_t(c+\rho)]}{c(\beta-\delta)}$ if

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[11] For this reason, y_n and y_τ are equal to the values found in the system without binding constraints.

[12] Others could be diffusion (e.g. through media), change of social norms like family size norms, etc.

[13] For instance, a continued increase of childrearing costs, as is commonly acknowledged across countries, leads to a continued reduction of the share of type 1. Furthermore, the old-age support becomes more difficult to afford for future generations.

$q_t < x_\tau$ and $C_{t+1}^1 = wBq_t$ if $q_t \geq x_\tau$, the consumption path is decreasing when q_t comes from $q_0 \approx 1$. Recall that the cultural process is starting from $q_0 \approx 1$, therefore the relevant area of analysis in figure 5 is from $q_0 \approx 1$ to the left. We observe that if the value of m is low, this never binds old-age consumption, so that y_τ will be the final distribution.

FIGURE 5: DISTRIBUTION OF TYPES AND INCREASE OF LIFE EXPECTANCY
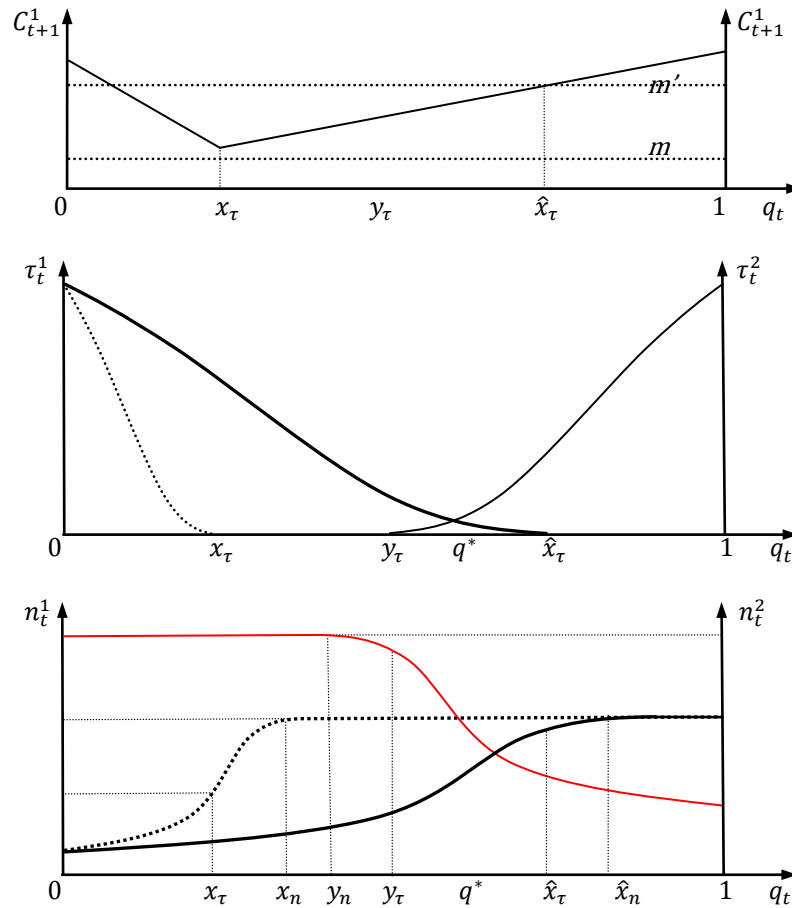
The final equilibrium of the system (q^*) will be different if m binds. We may consider that the minimum old-age consumption has been increasing over time due to the rise of life expectancy so that the line m moves up like in figure 5′s top panel[14]. To determine the location of this equilibrium, we must observe the middle panel. There, if the old-age consumption is not binding, the dotted and thin curves represent τ_t^1 and τ_t^2, respectively. At the point where the old-age consumption is binding $(q_\tau = \hat{x}_\tau)$, the socialization effort curve that is relevant for type 1 is not any more the dotted one, but the one from equation 19 represented by the bold line. Now, type 1 will cease his socialization effort only if there is a

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[14] In order to ease the explanation and construction of figure 5, we assume that type 2′s old-age consumption is not binding. Proposition 3 will consider fully the binding constraints for both types.

distribution of types at least as large as $\hat{x}_\tau$, which will allow her to reach the minimum old-age consumption. Therefore, if $y_\tau < \hat{x}_\tau$[15], then the equilibrium will be within the interval $(y_\tau, \hat{x}_\tau)$ as depicted in figure 5's middle panel. If $y_\tau \geq \hat{x}_\tau$, then the distribution of types will be y_τ as the system started from $q_0 \approx 1$.

Now, equilibrium $q^* \in (y_\tau, \hat{x}_\tau)$ implies a positive amount of τ_t^2 for type 2, but this is compensated by having fewer children. As figure 5's third panel depicts, type 2's fertility is computed at q^* instead of y_τ, which entails a lower fertility. Furthermore, type 1 must allocate more resources to socialization in equilibrium q^* than in equilibrium $\hat{x}_\tau$, although this is compensated by a reduction of fertility. This drop of fertility can also be observed in the third panel of figure 5: the relevant fertility curve for type 1 is now the bold curve (which represents equation 18) instead of the dotted one. At the new equilibrium q^*, type 1 fertility is lower. All these results are formalized by the next proposition.

Proposition 3: For a society with $q_0 \approx 1$ and $x_\tau \leq y_\tau$, a rise of life expectancy -expressed as an increase of minimum old-age consumption- leads to a distribution of types y_τ or $q^* \in (y_\tau, \hat{x}_\tau)$ if $y_\tau \geq \hat{x}_\tau$ and $y_\tau < \hat{x}_\tau$, respectively. Furthermore distribution q^* implies that socialization effort of both types is positive at equilibrium, and fertility declines overall.

Proof: see appendix.

Moreover, as a result of the previous proposition (provided that $y_\tau < \hat{x}_\tau$), the larger is m the larger is the share of type 1 in the distribution. This happens because equilibrium q^* is within the interval $(y_\tau, \hat{x}_\tau)$ and a rise of m also increase the value of $\hat{x}_\tau$, which in turn, enlarges the interval to the right. Thus, the society strengthens the values related to old-age family insurance as a reaction to the increase of life expectancy.

3.3 An illustration

The next example illustrates the dynamics of fertility and socialization. The parameter values are set in table 1 and the results are depicted in figure 6.

TABLE 1: PARAMETER VALUES

β	δ	ρ	c	R	B	m	w
0.99	0.3433	0.154	0.154	2.9126	0.35	0.30	1.0

Initial values:

q_0	n_0^1	τ_0^1	n_0^2	τ_0^2
0.95	6.1	0.0	2.07	0.88

---

[15] In parameter values, this means $\frac{c(1+\beta)}{c(1+\beta)+\rho} < \frac{m}{wB}$.

There is a wide range of assumed values for ρ in the literature of OLG models. For example this value is 0.075 in De la Croix and Doepke (2003) and the authors imply that a family could have up to 27 children. However, that number could not be reached because of biological limits. Bongaarts (1982) points out that, theoretically, the maximum number of children (births per woman) should be between 13 and 17. Accordingly, these limits are 6.5 and 8.5 children per person. We consider that 6.5 children per person is enough as an upper limit, so that we assume ρ=0.154[16]. We also assume that the cost of socialization effort is not different from the cost of rearing[17]. The discount factor value is assumed for convenience: under this value, type 2's old-age consumption is equal to m. Note that the implied yearly discount rate of δ is 4.37% if the adulthood period lasts 25 years, which is within other values assumed in OLG simulations (2% in Lau, 2009 and 4.7% in De la Croix and Doepke, 2003). For consistency, we assume this interest rate to compute the interest factor. As we set m=0.30, the reference transfer B is located above it at level 0.35. For all these parameters, the unconstrained consumption devoted to old-age period lies between 0.33 and 0.23, so that the minimum old-age consumption constraint will bind at some point.

FIGURE 6: PATHS TOWARDS EQUILIBRIUM
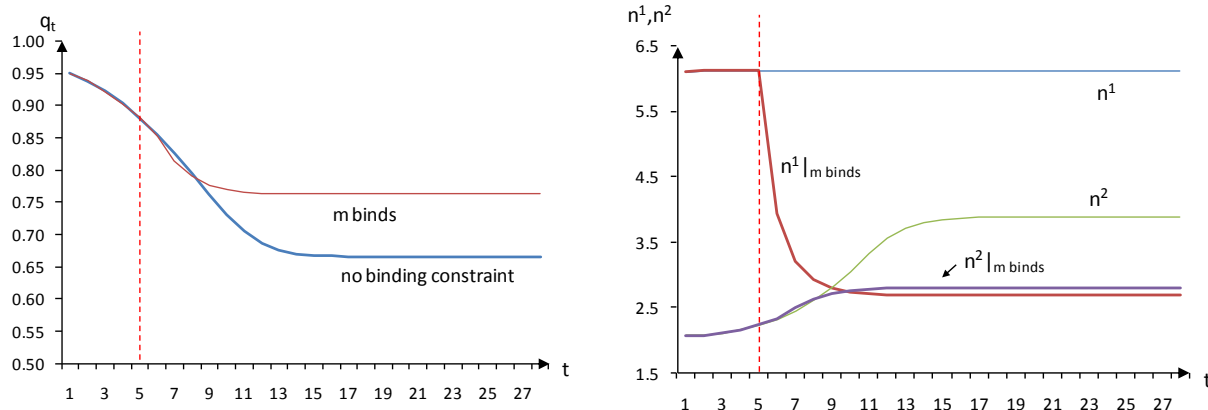
[16] According to the information from the Demographic and Health Surveys Program (http://www.measuredhs.com), the simple mean of the percentage of mothers with 10 or more children is only 1.65% for a pool of 13 Latin American countries.
[17] As rearing costs, activities related to socialization effort are also costly. Examples of these are the time spent with children, the search of a neighbourhood to reside in which the family finds more cultural homogeneity, and of the school to which the child is sent (Bisin and Verdier, 2001).
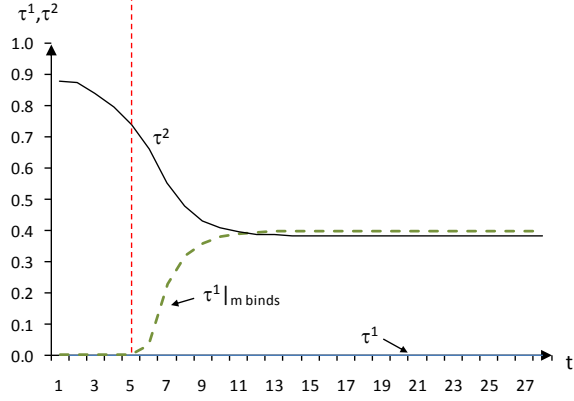
Figure 6 accords with the explanations stated in the previous section. If life expectancy does not play a role, the distribution of types would be y_τ=0.666 (see bold line of figure 6's top left panel) and the fertility rates of type 1 and 2 would be 6.12 and 3.88, respectively, which results in a weighted fertility rate of 5.37. However, in our simulation m binds from period 4, so that the final equilibrium is reached at q^*=0.762 and fertility of both types ends at lower levels (figure 6's top right panel). The overall fertility rate decreases to 2.71. Furthermore, from the point m starts to bind, type 1 increases her socialization (figure 6's bottom panel). Once the equilibrium is reached, both types end with positive values of socialization effort. In Table 2 we show the sensitivity of the final equilibrium with respect to changes in the parameter values.

TABLE 2: FINAL EQUILIBRIA FOR DIFFERENT PARAMETER VALUES

Parameter values		q^*	n^1	n^2	Overall n	τ^1	τ^2
baseline		0.762	2.69	2.80	2.71	0.40	0.38
ρ	0.175	0.736	2.29	2.48	2.34	0.46	0.42
	0.200	0.702	1.95	2.20	2.03	0.52	0.46
c	0.175	0.782	2.71	2.82	2.73	0.34	0.33
	0.200	0.800	2.75	2.86	2.77	0.29	0.28
β	0.800	0.734	2.44	2.61	2.49	0.46	0.43
	0.700	0.715	2.32	2.51	2.37	0.50	0.46

For instance, it is intuitive that an increase of ρ means that having children is more expensive relatively to socializing them, so that the fertility rate drops and the socialization effort rises. The opposite occurs when socialization cost increases. Furthermore, q^* drops when ρ rises as type 1 has fewer children and must devote more resources to socialization effort in order to meet consumption m in old-age. Type 2 also increases socialization effort but relatively less than type 1 because they do not need to exert a transfer from their

21

children. This prevents fertility rate of type 2 to fall more than type 1's; and at the end, the proportion of type 1 drops. If adults are less altruistic, they will have fewer children, so that socialization effort may increase.

Finally, table 3 highlights the consequences of an increase in life expectancy on the final equilibrium q^*. As mentioned in the previous section, a continued increase of life expectancy –measured through m- leads to a decline of overall fertility and a strengthening of type 1's values by a larger socialization effort. Type 2 also increases her socialization effort in order to face the increasing proportion of type 1 in society.

TABLE 3: FINAL EQUILIBRIA FOR DIFFERENT VALUES OF m

m	q^*	n^1	n^2	Overall n	τ^1	τ^2
0.300	0.762	2.69	2.80	2.71	0.40	0.38
0.305	0.772	2.60	2.73	2.63	0.44	0.42
0.310	0.782	2.52	2.66	2.55	0.48	0.45
0.315	0.793	2.44	2.59	2.47	0.52	0.49
0.320	0.805	2.36	2.53	2.39	0.56	0.52
0.325	0.818	2.27	2.46	2.31	0.61	0.56

As mentioned before, transfers from children would be insufficient to cover parent's minimum old-age consumption if m continues increasing such that $\frac{m}{Bw} > 1$. We might conjecture that if the old-age transfer does not increase, then old-age consumption falls short of subsistence level, which would lead to an increase of chronic poverty among the elderly. Alternatively, if the individual can only finance a fraction $\frac{Bw}{m}$ years of the total potential increase in life expectancy, this effect may also be interpreted as the inability of type 1 to take advantage from the exogenous increase in longevity. This suggests that there will be differentials in life expectancy once the individuals reach old-age, i.e. type 2 will live longer than type 1. Therefore, the position of type 2 in the distribution of types may become stronger in society through their higher proportion of elderly people, which reinforces the fall of the expected value of the old-age transfer. The continued fall of type 1's fertility (and reduction in the number of siblings) makes unbearable the payment of the old-age transfer, so that type 1 could eventually disappear. The same result is obtained even when the social norm of supporting elderly parents is strong enough to push for a rise of B in order to meet minimum old-age consumption. A rise of B allows the parent to fulfil minimum old-age consumption but also implies fewer resources for childbearing, which in turn reduces type 1's fertility. As B continues increasing to finance the increase of m, type 1's fertility falls considerably, even at the point where childbearing is not profitable. These conditions also

would jeopardize the existence of type 1. The next section shows some extensions to the model, which might moderate or delay the effects of the mentioned conjectures.

4. Extensions

In this section we still consider a society with characteristics $q_0 \approx 1$ and $x_\tau \leq y_\tau$ and analyse how our previous results change when i) the assumption of symmetric tolerance levels is relaxed and ii) a social security scheme is introduced.

4.1 Relaxing the assumption on the intolerance levels

Assumption 1 might be relaxed by allowing different levels of tolerance between types. We keep the normalization θ^{ii}=1 for i$\in$\{1,2\} and set $\theta^{12}, \theta^{21} \in [0,1]$. The higher θ^{ij} (with $i \neq j$), the more tolerant is each type towards the other. The program is easily solved for type 2 irrespective of whether the old-age consumption constraint is binding or not, but it is algebraically tedious for type 1 when the old-age consumption constraint is not binding. If this constraint binds for type 1, then θ^{12} does not play a role any more as type 1's socialization effort is entirely determined by the pursuit of meeting minimum consumption in old-age (see equation 19). As we intend to illustrate the sensitivity of our previous results, it is enough to consider values $\theta^{12} = 0$ and $\theta^{21} \neq 0$ to simulate the final equilibria (see table 4). This implies that type 2 is more tolerant to cultural deviation than type 1, which seems a reasonable assumption.

TABLE 4: FINAL EQUILIBRIA AND TOLERANCE LEVELS ($\boldsymbol{\theta^{12} = 0; \theta^{21} \neq 0}$)

θ^{21}	q^*	n^1	n^2	Overall n	τ^1	τ^2
0.00	0.762	2.69	2.80	2.71	0.40	0.38
0.05	0.785	2.84	2.93	2.86	0.34	0.33
0.10	0.807	3.04	3.09	3.05	0.26	0.25
0.15	0.829	3.31	3.32	3.31	0.17	0.17
0.20	0.848	3.70	3.66	3.69	0.06	0.06

One of the clear results is that parents socialize children less intensively towards their own type because cultural deviation is more tolerated. If type 2 parents are more tolerant, then they reduce their socialization effort, and consequently q^* increases. As q^* rises, type 1 parents need less investment in socialization effort. Therefore, socialization effort drops overall, which frees resources to invest in childbearing so that fertility rises overall as well. Although not shown in the table, a more tolerant environment, i.e. with a higher θ^{21}, delays

the time at which the old-age consumption constraint starts to bind. For example, in the simulation of section 3.3 that considers $\theta^{21} = 0$, m=0.30 begins to bind in period t=4. If the tolerance level is $\theta^{21} = 0.15$ or $\theta^{21} = 0.20$, the same value of m starts to bind at period t=6 and t=10, respectively. And for values $\theta^{21} > 0.2235$, m=0.30 will never bind. In sum, in a more tolerant society agents invest less in socialization effort and more in childbearing, and therefore the period at which children fail to be an adequate old-insurance is delayed.

4.2 Introduction of compulsory social security contribution

Developing countries where old-age family transfers are prominent may also have a pension system scheme. The characteristics are diverse: it can be PAYG, fully funded, based on individual accounts, compulsory, voluntary, etc. Therefore, it is interesting to assess the effects of a compulsory contribution to pensions on fertility, socialization and dynamics of our model. We will assume that a pension scheme based on individual capitalization is enforced in the model. This type of scheme is, for example, the most common in the Latin American countries. The pension contribution -a share of wages $\alpha \in [0,1)$- is paid during adulthood by all individuals and the returns αR are received during old-age, which is easily introduced into the system equations.

Old-age consumption is not binding

It is algebraically tedious to find the analytical solutions for type 1 when the old-age consumption constraint is not binding. However, the computation of type 1's first order conditions with different values of α allows us to conclude that fertility falls with the contribution rate[18]. As our interest is focused on societies with $q_0 \approx 1$ (recall that τ_t^1=0), the contribution rate will not affect socialization effort of type 1 when the old-age consumption restriction is not binding. Equations 23 and 24 describe the optimal responses for type 2:

$$n_t^2 = \begin{cases} \frac{q_t\beta}{(2+\delta+\beta)(q_t(\rho+c)-c)} & if\ \alpha \leq s_t\ ; q_t > y_n \\ \frac{q_t\beta(1-\alpha)}{(2+\beta)(q_t(\rho+c)-c)} & if\ \alpha > s_t\ ; q_t > y_n \\ \frac{(1-s_t)}{\rho} & if\ \alpha \leq s_t\ ; q_t \leq y_n \\ \frac{(1-\alpha)}{\rho} & if\ \alpha > s_t\ ; q_t \leq y_n \end{cases} \quad (23) \qquad \tau_t^2 = \begin{cases} \frac{q_t\rho-c(1+\beta)(1-q_t)}{cq_t\beta} & if\ q_t > y_\tau \\ 0 & if\ q_t \leq y_\tau \end{cases} \quad (24)$$

---

[18] The introduction of the social security contribution decreases fertility through two channels: there is an income effect due to less disposable resources for childbearing, and a substitution effect between αR and BP_t^{11}, which reduces the need of having children to fulfil elderly consumption.

The contribution rate will not affect directly type 2's fertility and socialization effort if $\alpha \leq s_t$. As the model is set (the voluntary old-age savings is equivalent to $wR(s_t - \alpha)$), an increase of α in period t will perfectly substitute the same quantity of personal old-age savings, and hence it should not have any effect on the outcome variables in period t. However, when we simulate the optimal path of type 2's socialization effort and fertility, we observe that a rise of α reduces socialization effort and increases fertility in the following periods. The reason is that –as mentioned- the increase of α lowers type 1' fertility, which reduces type 1' share in the total population. Therefore, type 2 devotes less time in socialization, which in turn frees resources to be allocated to fertility. If $\alpha > s_t$, the voluntary savings rate becomes zero, $wR\alpha$ being the total amount of savings; i.e. the individual is forced to save more than she would like. In this case, clearly an increase of α reduces type 2's fertility. The final effect of a change of α in period t on socialization in t+1 is undetermined.

Old-age consumption is binding

Equations 25 to 28 show the optimal fertility and socialization effort when old-age consumption restriction binds:

$$\hat{n}_t^1 = \begin{cases} \frac{Bw(1+\beta)(1-b_t-\alpha)(1-q_t)}{(2+\beta)\left(cm-Rcw\alpha+Bw(\rho-q_t(\rho+c))\right)} & if \ q_t \leq \tilde{x}_n \\[10pt] \frac{(1-b_t-\alpha)}{\rho} & if \ q_t > \tilde{x}_n \end{cases} \quad (25) \qquad \hat{\tau}_t^1 = \begin{cases} \frac{m-w(Bq_t+\alpha R)}{Bw(1-q_t)} & if \ q_t \leq \tilde{x}_\tau \\[10pt] 0 & if \ q_t > \tilde{x}_\tau \end{cases} \quad (26)$$

where $\tilde{x}_\tau = \frac{m-Rw\alpha}{Bw}$ and $\tilde{x}_n = \frac{(2+\beta)(cm-Rcw\alpha)+Bw\rho}{Bw(2c+\rho+c\beta)}$.

$$n_t^2 = \begin{cases} \frac{q_t\beta(Rw-m)}{Rw(2+\beta)(q_t(\rho+c)-c)} & if \ \alpha \leq s_t \ ; q_t > y_n \\[8pt] \frac{q_t\beta(1-\alpha)}{(2+\beta)(q_t(\rho+c)-c)} & if \ \alpha > s_t \ ; q_t > y_n \\[8pt] \frac{(1-s_t)}{\rho} & if \ \alpha \leq s_t \ ; q_t \leq y_n \\[8pt] \frac{(1-\alpha)}{\rho} & if \ \alpha > s_t \ ; q_t \leq y_n \end{cases} \quad (27) \qquad \tau_t^2 = \begin{cases} \frac{q_t\rho-c(1+\beta)(1-q_t)}{cq_t\beta} & if \ q_t > y_\tau \\[8pt] 0 & if \ q_t \leq y_\tau \end{cases} \quad (28)$$

In general, standard models of fertility and growth predict that the increase of social security may lower fertility (Becker and Barro, 1988; Boldrin and Jones, 2002). For instance, Barro and Becker (1988) treat the increase of social security contributions and benefits as an increase of the cost of rearing a child, so that fertility should drop –at least temporarily- due

to a rise of social security. In our model we also observe that the fertility (of type 1) is negatively related to the level of the contribution rate when m is not binding. However, when m binds, $\frac{\partial \hat{n}_t^1}{\partial \alpha} = \frac{Bw(1-q_t)(1+\beta)\left(Rcw(1-b_t)-cm-Bw\rho+q_tBw(c+\rho)\right)}{(2+\beta)\left(Rcw\alpha-cm-Bw\rho+q_tBw(c+\rho)\right)^2}$, so that the effect of a rise of the contribution rate on type 1's fertility depends on the parameter values. After computing this derivative –through simulations- for different values of α, we find that in general $\frac{\partial \hat{n}_t^1}{\partial \alpha}$ is positive if α is small but it is negative for higher levels of α. This -apparently counter-intuitive- result must be interpreted within the framework of our model. Recall that the old-age consumption of type 1 is bounded by m, which pushes down their fertility rate and triggers up their socialization effort. Socialization effort decreases with the contribution rate because the parent is forced to save αw, which reduces the transfer amount that she needs from her children in order to consume exactly m in old-age. Therefore, an increase of α reduces the need to socialize children, so that some resources may be freed. These resources might be invested in having more children or consuming more during the adulthood.

In table 5, we illustrate the effects of different contribution and interest rates on the equilibrium values. We assume the same parameters as in the previous exercises. The effect of the interest rate on the fertility of type 1 depends on the value of α. For α=0, this effect is negative but it is positive when α>0. For type 2, this effect is positive. The interest rate is negatively related to the socialization effort of type 2 and type 1 (when α>0). Overall, we observe that the fertility rate increases with the interest rate (as in the model of Becker and Barro, 1988).

TABLE 5: FINAL EQUILIBRIA AND CONTRIBUTION RATE

α	q^*	n^1	n^2	Overall n	τ^1	τ^2
if annual interest rate=4.37%						
0.00	0.762	2.69	2.80	2.71	0.40	0.38
0.01	0.714	3.14	3.22	3.16	0.21	0.20
0.02	0.676	3.65	3.71	3.67	0.05	0.05
if annual interest rate=2.00%						
0.00	0.771	2.74	2.50	2.68	0.38	0.41
0.01	0.738	2.96	2.72	2.89	0.27	0.30
0.02	0.711	3.19	2.96	3.12	0.18	0.19

If we keep the values of all parameters of the baseline simulation (table 1) except m, and set that old-age consumption is binding, we can plot a path of final equilibria for each value of m. This is shown in figure 7's left panel. The horizontal axis may be interpreted as life expectancy, so that the equilibria are the optimal responses to a continued increase in life

expectancy. If there is no social security (α=0.0 and old-age transfer remains constant, B=0.35), then type 1 is unable to take advantage from the exogenous increase of life expectancy once the point m=0.35 is reached (indicated by the peak in figure 7). From that point, type 1 can only finance a fraction $\frac{Bw}{m}$ years of the total potential increase in life expectancy (as conjectured in section 3.3); meanwhile, her fertility continues falling as is observed in the right panel of figure 7. At that point and later, type 2 can reallocate more resources from socialization to childrearing and increases fertility as q^* is falling. The reduction of the share of type 1 in the distribution is reinforced by the rise of type 2's fertility and the fact that the expected old-age transfers go down (which triggers τ^1 up)[19]. Eventually, the number of type 1's children may be too low to cope with the burden of old-age support to parents, so that at some point type 1 could stop having children and disappear, provided that B or other variables remain unchanged. However, the introduction of social security might mitigate this effect as can be observed in figure 7. For instance, a contribution rate of 0.02 may delay the declining of the type 1's share in the distribution up to the point where m=0.408 (see the peaks of the dotted lines in figure 7). Furthermore, if there is no social security, reproduction of type 1 may stop at m=0.4168 (q^*=0.76). But, if α=0.02, type 1 extinguishes at 0.4675 (q^*=0.711).

FIGURE 7: FINAL EQUILIBRIA FOR DIFFERENT LEVELS OF m
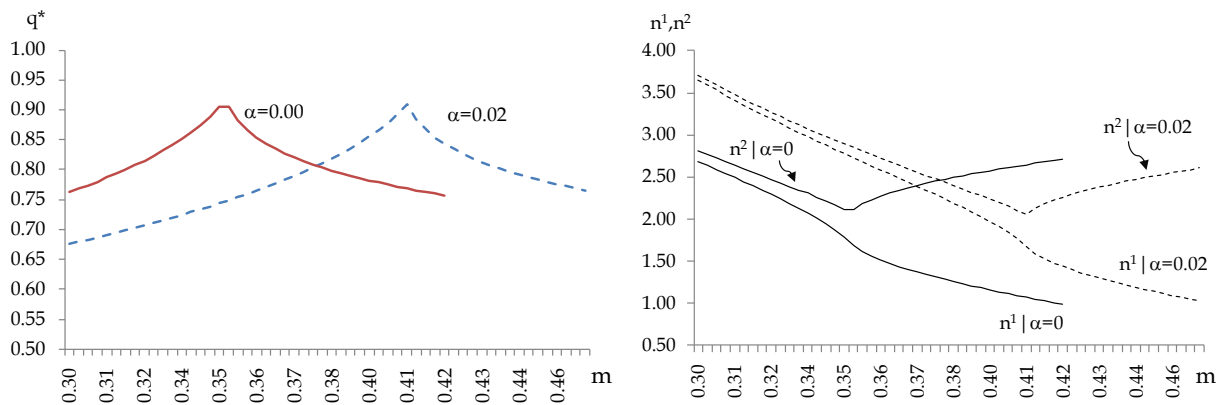
4.3 Trends and demographic projections in Latin American countries

Demographic trends in Latin American countries indicate a substantial fall of fertility and increase of life expectancy. Given that these economies show a prominent role for old-age

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[19] Although not shown in figure 7, the overall fertility declines.

family transfers, one could think that the structural pension reform undertaken during the 90's might help the population to cope with the old-age risk, but this is not really true as the rates of coverage are low (38.7% of employed). These rates are as low as 13.0% in Peru at the national level (2.6% in rural areas) or as high as 65.3% in Costa Rica. Furthermore, poverty rates are particularly high in rural areas; there, type 1 values are more important and hence the budget of the families is very likely to be bounded by subsistence consumption[20].

The introduction and strengthening of compulsory social security theoretically enhances old-age consumption and help for the sustainability of type 1 in the long run, but it is observed that in practice only salaried workers from the formal sector contribute to pensions. The informal sector may be huge and diverse, ranging from 30.7% of the labour force in Chile to 64.6% in Peru; 49.2% being the simple average. An easy way to observe how the informal sector size reduces the effectiveness of social security is by adding π, the probability of belonging to the informal sector, into the consumption constraints of the model[21]:

$$C_t^1 = w_t(1 - n_t^1(\rho + c\tau_t^1) - b_t - \alpha\pi) \text{ and } C_{t+1}^1 = w_{t+1}(BP_t^{11} + \alpha\pi R)$$
$$C_t^2 = w_t(1 - n_t^2(\rho + c\tau_t^2) - s_t - \alpha\pi) \text{ and } C_{t+1}^2 = w_t R(s_t + \alpha\pi)$$

Now, the consumption is a weighted sum of the consumption obtained in the formal sector (paying α) and in the informal sector (α=0). Clearly, a larger informal sector is equivalent to a lower contribution rate.

As pointed, the decline of fertility and rise of life expectancy threatens the ability of type 1 to afford minimum old-age consumption, and even their existence in the long run. The Governments of Latin American countries cannot rely on the support of children to parents as a permanent way to face the ageing of population. Hence, it is understandable that a pension system is launched as a way to help agents to obtain a certain level of consumption in old-age. However, informality might reduce the scope of this system through a lower expected contribution rate. Although our model does not distinguish individuals according to their occupation in the formal or informal sector, salaries or other socio-economics characteristics, it is possible to incorporate them and evaluate more precisely the relations

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[20] All these figures come from the database of the Economic Commission for Latin American and the Caribbean.

[21] This may be interpreted as once the individual embraces a type, she is randomly assigned to the informal sector with probability π. Of course, probability distributions of types and informality are not necessarily independent, but assuming the contrary would add unnecessary complexity.

found within the framework of a country with a high share of informality. Note that workers earn lower salaries in the informal sector, so that minimum consumption will bind their budgets more easily than that of formal workers. An additional option is a change of the cultural tolerance levels of society, although this is hardly applicable from a policy intervention point of view.

5. Conclusions

Cultural models of family matter for the dynamics of fertility. Equipped with a cultural transmission technology, our model can account for the demographic change by showing how the increase of life expectancy may reduce fertility and the expected old-age transfer from children through a strengthening of cultural socialization efforts. While we analyse the long run equilibria, we conjecture that a continued increase of life expectancy can jeopardise the existence of the old-age supporter trait. This is a well cast example of non efficient cultural reaction to the economic environment. Furthermore, we have shown that societies becoming more tolerant or the introduction of compulsory social security might improve the ability of children to support parents and counter the extinction of the old-age supporters. Finally, our model may highlight some of the future demographic pressures and the possible deterioration of the standard of living of the elderly in Latin American countries.

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Appendix

Proof of proposition 1:

The dynamic is governed by $\Delta q_t = q_{t+1} - q_t = \frac{q_t(1-q_t)(n_t^1\tau_t^1 - n_t^2\tau_t^2)}{1+n_t^2+q_t(n_t^1-n_t^2)}$. In this system, the steady

state q^* is locally stable is $\frac{\partial \Delta q_t}{\partial q_t}\Big|_{q^*} < 0$. First we check equilibria 0 and 1:

For equilibrium $q^*=0$, we have $\tau_t^2 = 0$ and $n_t^2 = \frac{2+\beta}{(2+\beta+\delta)\rho}$. Thus,

$$\Delta q_t = \frac{\frac{m_1(1-q/x_\tau)}{m_5-q}}{\frac{1}{q(1-q)}+\frac{m_6}{q}+\frac{m_4}{m_5-q}} \tag{A1}$$

where $m_1 = \frac{\rho(1+\delta)(1-b)}{c(2+\beta)(\rho+c)}$; $m_4 = \frac{(\beta-\delta)(1-b)}{(2+\beta)(\rho+c)}$; $m_5 = \frac{\rho}{(\rho+c)}$ and $m_6 = \frac{2+\beta}{(2+\delta+\beta)\rho}$.

As $\frac{\partial \Delta q_t}{\partial q_t}\Big|_{q^*=0} = \frac{m_1(1+m_6)}{m_5} > 0$, the equilibrium $q^*=0$ is unstable.

For equilibrium $q^*=1$, we have $\tau_t^1 = 0$ and $n_t^1 = \frac{1-b}{\rho}$. Thus,

$$\Delta q_t = \frac{m_2(1-q/y_\tau)}{\frac{q-(1-m_5)}{q(1-q)}+m_3+\frac{m_7(q-(1-m_5))}{1-q}} \tag{A2}$$

where $m_2 = \frac{1+\beta}{(2+\delta+\beta)(\rho+c)}$; $m_3 = \frac{\beta}{(2+\delta+\beta)(\rho+c)}$ and $m_7 = \frac{1-b}{\rho}$.

$\frac{\partial \Delta q_t}{\partial q_t}\Big|_{q^*=1} = \frac{m_2 m_5(1+m_7)(1-y_\tau)}{y_\tau(m_5+m_7 m_5)^2} > 0$, so that the equilibrium $q^*=1$ is unstable.

Now, we distinguish two cases of different parameter values:

a) $x_\tau \leq y_\tau$:

If initial condition is $q_0 \approx 1$, then $\tau_t^1 = 0$ and $\tau_t^2 > 0$, and hence the dynamics stops at equilibrium $q^* = y_\tau$. Similarly, if initial condition is $q_0 \approx 0$, then $\tau_t^2 = 0$, $\tau_t^1 > 0$ and the equilibrium is $q^* = x_\tau$. If q_0 is initially located within the range (x_τ, y_τ), then there is no motion in the system and hence q_t remains at q_0.

a.1) If $q^* \in (x_\tau, y_\tau)$:

If q_0 is initially located within the range (x_τ, y_τ), then there is no motion in the system and hence q_t remains at $q_0 = q^*$. For any small disturbance $y_\tau - q^* > e < q^* - x_\tau$, the system does not go back to q* because there is no motion within (x_τ, y_τ) and hence q^* is unstable.

a.2) If $q^* = x_\tau$:

We evaluate equation A1 at $q^*=x_\tau$:

$$\frac{\partial \Delta q_t}{\partial q_t}\Big|_{q^*=x_\tau} = -\left(\frac{m_1}{m_5-x_\tau}\right)\frac{1-x_\tau}{1+m_6(1-x_\tau)+\frac{m_4 x_\tau(1-x_\tau)}{m_5-x_\tau}} < 0$$

a.3) If $q^* = y_\tau$:

We evaluate equation A2 at $q^*=y_\tau$:

$$\frac{\partial \Delta q_t}{\partial q_t}\Big|_{q^*=y_\tau} = -\frac{m_2(1-y_\tau)}{m_3 y_\tau(1-y_\tau)+(1+m_7 y_\tau)(y_\tau-(1-m_5))} < 0$$

a.4) If $q^* = y_\tau = x_\tau$

As $q^* = y_\tau = x_\tau$, a small disturbance in the boundary of q^* may lead to positive socialization effort, thus we construct Δq_t with $\tau_t^1 > 0$ and $\tau_t^2 > 0$. Furthermore, $n_t^1 = \frac{(1-b_t)(\beta-\delta)(1-q_t)}{(2+\beta)(\rho-q_t(\rho+c))}$ and $n_t^2 = \frac{q_t\beta}{(2+\delta+\beta)(q_t(\rho+c)-c)}$:

$$\Delta q_t = \frac{\frac{m_1(1-q/x_\tau)}{m_5-q}+\frac{m_2(1-q/y_\tau)}{m_5-(1-q)}}{\frac{1}{q(1-q)}+\frac{m_3}{m_5-(1-q)}+\frac{m_4}{m_5-q}} \tag{A3}$$

And $\frac{\partial \Delta q_t}{\partial q_t}\Big|_{q^*=x_\tau=y_\tau} = \frac{-\frac{1}{q^*}\left(\frac{m_1}{m_5-q^*}+\frac{m_2}{m_5-(1-q^*)}\right)}{\frac{1}{q^*(1-q^*)}+\frac{m_3}{m_5-(1-q^*)}+\frac{m_4}{m_5-q^*}} < 0$, so that q^* is stable.

b) $x_\tau > y_\tau$:

The steady state q^* is obtained from solving $\Delta q_t = 0$ in equation A3, i.e.: $\frac{m_1(1-q/x_\tau)}{m_5-q} + \frac{m_2(1-q/y_\tau)}{m_5-(1-q)} = 0$. We evaluate stability of q^* by taking the derivative of equation A3:

$$\frac{\partial \Delta q_t}{\partial q_t} = \frac{\left(\frac{1}{q(1-q)}+\frac{m_3}{m_5-(1-q)}+\frac{m_4}{m_5-q}\right)W-\left(\frac{m_1(1-q/x_\tau)}{m_5-q}+\frac{m_2(1-q/y_\tau)}{m_5-(1-q)}\right)X}{\left(\frac{1}{q(1-q)}+\frac{m_3}{m_5-(1-q)}+\frac{m_4}{m_5-q}\right)^2} \tag{A4}$$

where $W = \frac{m_1}{(m_5-q)x_\tau}\left(\frac{x_\tau-q}{m_5-q}-1\right)-\left(\frac{m_2}{y_\tau(m_5-(1-q))}\right)\left(\frac{y_\tau-q}{(m_5-(1-q))}+1\right)$ and $X = \frac{2q-1}{q^2(1-q)^2} - \frac{m_3}{(m_5-(1-q))^2} + \frac{m_4}{(m_5-q)^2}$.

If $x_\tau > y_\tau$, then $q^* < x_\tau$ and $q^* > y_\tau$; and hence $W<0$. The sign of X is undetermined. However, the expression multiplying X in A4 should be zero as this is the condition to find equilibrium q^*. Furthermore, the expression multiplying W in A4 is positive due to $m_5 > x_\tau > q^* > y_\tau > 1 - m_5$. Therefore, $\frac{\partial \Delta q_t}{\partial q_t}\Big|_{q^*} = \frac{(+)(W)-(0)(X)}{(+)^2} < 0$ and q^* is stable.

For this q^* we have inspected -with simulations- that the system undergoes a bifurcation near $c^* = \frac{\rho\sqrt{1+\delta}}{1+\beta}$. When c passes this threshold (i.e. $c<c^*$), the system initially undergoes a period-doubling bifurcation, and then it becomes chaotic and hard to analyze (like in figure 3). This chaotic behaviour is produced by the dynamics summarized in equation 12, which is a non-linear second order difference equation (and with kinks). Therefore, we can only state that q* is locally stable.

Proof of proposition 2:

a) Provided that $\hat{q}_t < x_\tau$, we can find the distribution of types $\hat{q}_t$ at which the old-age consumption binds: $C_{t+1}^1 = \frac{wB(1+\delta)(\rho - \hat{q}_t(c+\rho))}{c(\beta-\delta)} = m$; and this is $\hat{q}_t = \frac{Bw\rho(1+\delta) - cm(\beta-\delta)}{Bw(1+\delta)(c+\rho)}$. We replace $\hat{q}_t$ into equation 18 (recall that $x_n > x_\tau$ and $b_t = B/n_{t-1}^1$) and obtain optimal fertility $\hat{n}_t^1 = \frac{(n_{t-1}-B)(m(\beta-\delta)+Bw(1+\delta))}{mn_{t-1}(2+\beta)(c+\rho)}$. The sign of $\frac{\partial \hat{n}_t^1}{\partial B}$ is not conclusive as $\frac{\partial \hat{n}_t^1}{\partial B} = \frac{(n_{t-1}-2B)w(1+\delta)-m(\beta-\delta)}{mn_{t-1}(2+\beta)(c+\rho)}$. If we define threshold $z = \frac{n_{t-1}w(1+\delta)-m(\beta-\delta)}{2w(1+\delta)}$, then $\frac{\partial \hat{n}_t^1}{\partial B} > 0$ if $B < z$; and $\frac{\partial \hat{n}_t^1}{\partial B} \leq 0$ if $B \geq z$.

b) Provided that $\hat{q}_t \geq x_\tau$, then $C_{t+1}^1 = wB\hat{q}_t = m$, so that $\hat{q}_t = \frac{m}{Bw}$. Therefore, $\hat{n}_t^1 = \frac{(1+\beta)(1-b_t)}{\rho(2+\beta)}$ and $\frac{\partial \hat{n}_t^1}{\partial B} < 0$.

Proof of proposition 3:

a) Distribution when $y_\tau \geq \hat{x}_\tau$:

We know that m binds type 1´s consumption at $q_t = \hat{x}_\tau$; m can also bind type 2´s consumption or not. If m binds type 2's consumption, then the steady state $q^* = y_\tau$ is obtained from $\Delta q_t = 0$ ($\equiv -\hat{n}_t^2 \hat{\tau}_t^2 = 0$) provided that $q_0 \approx 1$ and $y_\tau \geq \hat{x}_\tau$. Similarly, $q^* = y_\tau$ is also the steady state when m does not bind type 2's consumption (solving $-n_t^2 \tau_t^2 = 0$).

b) Distribution when $y_\tau < \hat{x}_\tau$:

We proof that $q^* \in (y_\tau, x_\tau)$ when $y_\tau < \hat{x}_\tau$ by contradiction. We know that m binds type 1´s consumption at $q_t = \hat{x}_\tau$. Assume that m does not bind type 2´s consumption. If $\hat{\tau}_t^1 = 0$, $q^* = \frac{c(1+\beta)}{c(1+\beta)+\rho} = y_\tau$; and if $\tau_t^2 = 0$, $q^* = \frac{m}{Bw} = \hat{x}_\tau$. But, given that we are assuming $y_\tau < \hat{x}_\tau$, these values are a contradiction as $\hat{\tau}_t^1$ and τ_t^2 are zero only at $q_t \geq \hat{x}_\tau$ and $q_t \leq y_\tau$, respectively. Therefore, $\tau_t^1, \tau_t^2 \neq 0$ and consequently $q^* \in (y_\tau, x_\tau)$. The same results apply if m also binds type 2´s consumption.

When m binds, the fertility differential for type 2 is $\hat{n}_t^2 - n_t^2 = \frac{-q_t\beta(m(2+\beta+\delta)-Rw\delta)}{Rw(2+\beta)(2+\beta+\delta)(q_t(\rho+c)-c)}$. This expression is negative if $\frac{\delta Rw}{2+\delta+\beta} < m$, which holds given that $\frac{\delta Rw}{2+\delta+\beta} < m$ is equivalent to $C_{t+1}^2 < m$. The type 1´s fertility differential is $\hat{n}_t^1 - n_t^1 = \frac{1-b_t}{2+\beta}\left(\frac{1+\beta}{\rho} - \frac{(\beta-\delta)(wB-m)}{\rho(wB-m)-cm}\right)$ when m binds (at $\hat{x}_\tau = \frac{m}{wB}$). This expression is negative if $\frac{\rho(1+\delta)}{c(1+\beta)+\rho(1+\delta)} < \frac{m}{wB}$, which holds given that

this is equivalent to $x_\tau < \hat{x}_\tau$. Recall we are evaluating steady states $q^* \in (y_\tau, \hat{x}_\tau)$ and that x_τ must be lower than $\hat{x}_\tau$ as we assume dynamics with $y_\tau \geq x_\tau$ and $q_0 \approx 1$.

Therefore, fertility falls overall when equilibrium is $q^* \in (y_\tau, \hat{x}_\tau)$. From that point, it is clear that $\hat{n}_t^1$ and $\hat{n}_t^2$ depend negatively on m.